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Fibrous Products (5 cpu / 3.5 ov) 3513057

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- General overview
- Specific Knowledge
  - no focus in technical detail
  - Focus in Core processes and mechanisms+
    - fiber processing
    - forming and consolidation
    - structure and properties of products

Information Technology:

- language, speech
- drawings on rock walls
- clay boards
- parchment
- papyrus (first Dynasty, 3000 BC)
- four great Inventions of Ancient China:
  - compass
  - gunpowder
  - paper – 8 BC
  - printing
- -> handmade paper to Europe 13'th Century (hemp and linen rags)

Gutemberg printhouse 1450
Nicholas-Luis Robert: Furdinier paper machine 1807
1830's: Friedrich Keller
Charles Fenerty Grinding wood into pulp

Knut Fredrik Idestam:
Groundwood pulp mill at Tampere 1866
-> Emäkoski rapids at Nokia 1869
Nokia Aktiebolaget 1871

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Porous sheet properties

Basis Weight Grammage

Porosity

Solid Fraction

Density

Fiber Coverage

Fiber Geometry (shape)

Aspect Ratio $\frac{l}{w}$

Slenderness $\frac{\pi l}{p}$

Oblongness $\frac{\pi l^2}{4A}$

Raggedness $\frac{p^2}{4\pi A}$

Compactness $\frac{4\pi A}{p^2}$

Fiber size

Thickness

Width

Length

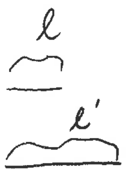
Mass

Coarseness

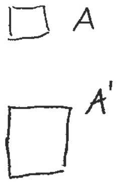
2-d fiber network connectivity

(Basis Weight * Fiber length) / Coarseness

TRIVIAL SCALING



$$l' = q\,l$$



$$A' = q^2 A$$

$$V' = q^3 V$$

$$m' = q^3 m$$

$$\frac{A'}{m'} = \frac{q^2 A}{q^3 m} = q^{-1}\frac{A}{m}$$

$$2^{-1}\frac{A}{m} = \frac{1}{2}\frac{A}{m}$$

RBA

Bonded Area $= A_f \cdot RBA$

Free Area $= A_f\,(1-RBA)$

$$\frac{\text{Free Area}}{\text{mass}} = \frac{N_f}{m} A_f (1-RBA) = \frac{m}{m_f\, m} A_f (1-RBA)$$

$$= \frac{A_f}{m_f}(1-RBA)$$

$$\approx \frac{2}{\rho_f\, r}(1-RBA)$$

$$A_f \propto r^2 \approx 2\pi r^2 \frac{l}{r}$$

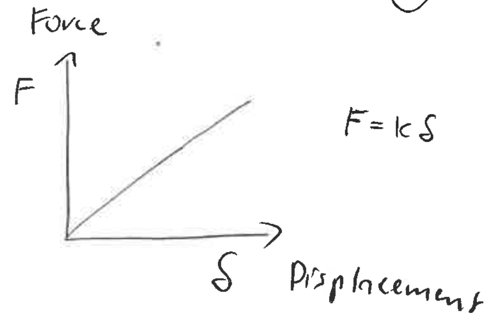
$$m_f = \rho_f V_f \propto r^3 \approx \rho_f \pi r^3 \frac{l}{r}$$

$\Rightarrow$ Filler capacity $\propto r^{-1}$

or

$$\frac{\left(\frac{\text{Free Area}}{\text{mass}}\right)_2}{\left(\frac{\text{Free Area}}{\text{mass}}\right)_1} = \frac{r_1}{r_2}$$

Stiffness and Strength



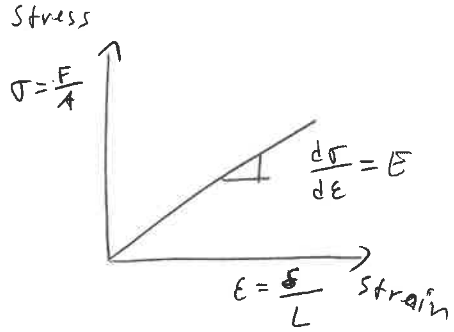
$F = k\delta$

Fiber Stress (average)

$$\langle \sigma_f \rangle = E_f \langle \varepsilon_f \rangle$$

Paper Stress

$$\sigma_p = E_p \varepsilon_p$$



Forces in paper transmitted through Fibers

$$\bar{u} \cdot \bar{F}_p = \sum \bar{u} \cdot \bar{F}_f$$

$$\bar{F}_p = \sum \cos\theta \, \bar{F}_f = E_f \sum \cos\theta \langle \varepsilon_f \rangle A_f$$

$$\sigma_p A_p =$$

$$\langle \varepsilon_f \rangle = \mu \cos\theta \, \varepsilon_p$$

$$E_p \varepsilon_p A_p = E_f \sum \cos\theta \langle \varepsilon_f \rangle A_f$$

$$= E_f \sum \cos^2\theta \, \mu \, \varepsilon_p A_f$$

$$E_p = E_f \sum \cos^2\theta \, \mu \frac{A_f}{A_p} = E_f (1-P) \sum \cos^2\theta \, \mu$$

$$= E_f \cdot \text{Dimensionless}$$

$\Rightarrow$ Independent of Fiber size!

STRENGTH: $\sigma_c = \varepsilon_c^e E =$

$$\sigma_{cp} = \varepsilon_{cp}^e E_p = \text{Dimensionless} \cdot E_f \quad !$$

Fiber Size $\rightarrow$ Basis Weight at Constant Fiber Coverage

$$Bw = \frac{m}{A} = \frac{\rho V}{A} = \frac{\rho t A}{A} = \rho t$$

$$Bw = \frac{m}{A} = \frac{\sum m_f}{A} = \frac{n_f m_f}{A} = \frac{n_f \rho_f V_f}{A} = \frac{n_f w_f l_f t_f \rho_f}{A}$$

$$= C \, t_f \rho_f$$

$$= C \, Bw_f$$

$$C = \text{Fiber Coverage} = \frac{n_f w_f l_f}{A}$$

Bw_f = Fiber Basis Weight

$$\equiv \frac{m_f}{A_=} = \frac{m_f}{w_f l_f} = \frac{C_f}{w_f} = \rho_f t_f$$

$$C_f = \text{Coarseness} \equiv \frac{m_f}{l_f}$$

Trivial Scaling $\quad S' = q^{\mu_s} S$

Trivial Scaling of Fiber Basis Weight

$$Bw_f' = q^{\mu} Bw_f \qquad \text{What is scaling exponent } \mu?$$

$$Bw_f' = \left(\frac{m_f}{A_=}\right)' = \frac{m_f'}{A_=' } \qquad m_f' = (\rho_f V_f)' = \rho_f q^3 V_f = q^3 m_f$$

$$A_=' = q^2 A_=$$

$$Bw_f' = \frac{q^3}{q^2} Bw_f = q^1 Bw_f$$

Determination of FSP through Solute Exclusion Technique

1° Produce a solution of molecules, concentration $c_1 = \frac{m_1}{V_1}$

2° Add wet porous substance, mass of solids in relation to volume of water $c_2 = \frac{m_2}{V_2}$

3° some of the water coming with the substance dilutes the solution, concentration becomes

$$c_3 = \frac{m_1}{V_3}$$

What is now V_3 ?

That is water volume accessible to the molecules. $V_1 + V_2 = V_3 + V_4$

V_4 is Inaccessible water volume

$$V_4 = V_1 + V_2 - V_3 = \frac{m_1}{c_1} + \frac{m_2}{c_2} - \frac{m_1}{c_3}$$

$$FSP \left[\frac{(\,)}{(\,)}\right] = \frac{V_4 \cdot \rho_w}{m_2} = \rho_w \left[\frac{m_1}{m_2}\left(\frac{1}{c_1} - \frac{1}{c_3}\right) + \frac{1}{c_2}\right]$$

$V_4 \rho_w$ = mass of water in pores Inaccessible to molecules

Which other ways do we have for determining cell wall porosity?

Let us discuss some thermodynamic potentials:

Internal Energy

$$U = \underset{\text{Heat}}{TS} \; \underset{\text{work}}{-pV} \; \underset{\text{Chem. pot.}}{+\mu N} \qquad dU = TdS - pdV + \mu dN$$

Enthalpy

$$H \equiv U + pV = TS + \mu N \qquad dH = TdS + Vdp + \mu dN$$

Gibb's Function

$$G \equiv U - TS + pV = \mu N \qquad dG = -SdT + Vdp + \mu dN$$

Phase Transition: $dp = dT = 0$

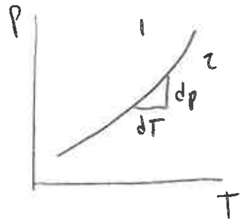
$$\Rightarrow \Delta H = T\Delta S$$

Change of Heat

Coexistense: $G_1(p, T, N) = G_2(p, T, N)$

$\Delta G_2 - \Delta G_1 = 0$

$$\Delta G_1 = -S_1 dT + V_1 dp$$
$$\Delta G_2 = -S_2 dT + V_2 dp$$



$$-(S_2 - S_1)dT + (V_2 - V_1)dp = 0$$

$$\frac{dp}{dT} = \frac{\Delta S}{\Delta V} = \frac{\Delta H}{T\Delta V} \qquad \text{Clausius – Clapeyron Eq.}$$

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Surface Energy γA

$$F = \frac{dW}{d\delta} = \frac{d(\gamma A)}{dr} = \frac{d\ \gamma 4\pi r^2}{dr} = 8\pi r\gamma$$

Stress due to surface tension

$$\frac{F}{A} = \frac{8\pi r\gamma}{4\pi r^2} = \frac{2\gamma}{r}$$

Balance of Forces

$$p_{in} 4\pi r^2 = p_{out} 4\pi r^2 + 8\pi r\gamma$$

$$\Delta p = p_{in} - p_{out} = \frac{2\gamma}{r}$$ Laplace Eq.

Molar Gibbs Function

$$G_m = \frac{G}{n} = \mu N_A$$

$$dG_m = -\frac{S}{n}dT + \frac{V}{n}dp + \frac{\mu}{n}dN$$

$$= -S_m dT + V_m dp + \mu_m dN$$

Equilibrium of liquid w. Ideal Gas

$$\mu_g = \mu_\ell \Rightarrow G_{mg} = G_{m\ell}$$

at $dT = dN = 0$

$$V_{mg}\, dp_g = V_{m\ell}\, dp = V_{m\ell}\left(dp(r=\infty) + d\Delta p\right)$$

Ideal Gas: $pV_m = RT$

$$\frac{RT}{p} dp_g = V_{m\ell}\, dp_\infty + V_{m\ell}\, d\Delta p$$

$$RT \ln p_g = V_{m\ell}\, p_\infty + V_{m\ell}\,\Delta p + C$$

$$= V_{m\ell}\, p_\infty + V_{m\ell}\frac{2\gamma}{r} + C$$

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$$p_g = e^{\frac{V_{m\ell} p_\infty}{RT}}\, e^{\frac{V_{m\ell} 2\gamma}{rRT}}\, C_2$$

$$= C_3\, e^{\frac{V_{m\ell} 2\gamma}{rRT}}$$

$$= p_\infty\, e^{\frac{V_{m\ell} 2\gamma}{rRT}}$$

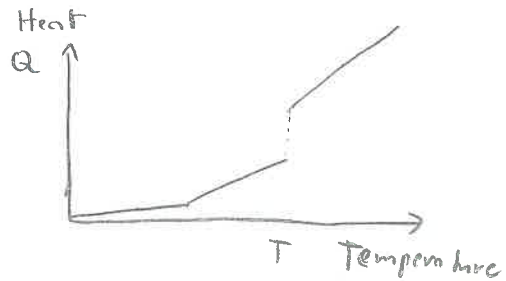
$$= p_\infty\, e^{\frac{2\gamma M}{rRT\rho}}$$ KELVIN Eq.

$$\begin{cases} r \to \infty \Rightarrow \\ e^{\frac{1}{r}} \to 1, \quad p_g \to p_\infty \end{cases}$$

Set $p_g(r=r_s) = p_s$

$$p_s = p_\infty e^{\frac{2\gamma M}{r_s RT\rho}} \Rightarrow p_\infty/p_s = e^{-\frac{2\gamma M}{r_s RT\rho}}$$

Thermal transitions



Heat Capacity $\frac{dQ}{dT}$

First-order Transition

Second-order Transition

Melting Temperature Spectrum

Clausius-Clapeyron

$$\frac{dp}{dT} = \frac{\Delta H}{T\Delta V} < 0$$

$$dp = \frac{\Delta H}{T\Delta V} dT$$

$$p = \frac{\Delta H}{\Delta V}\ln T + C$$

$$T = C_2\, e^{\frac{p\Delta V}{\Delta H}} = C_2\, e^{-p\left|\frac{\Delta V}{\Delta H}\right|}$$

Melting Temperature Spectrum

Coexistence of Solid and Liquid

Chemical potential $\mu = \frac{G}{N}$ must be equal

$$d\mu^s = d\mu^\ell$$

$$dG^s = dG^\ell$$

$$-S^s dT + V^s dp^s = -S^\ell dT + V^\ell dp^\ell$$

$$(S^s - S^\ell)\, dT = V^s dp^s - V^\ell dp^\ell$$

$$-\Delta S\, dT = V^s d(p^\ell + \Delta p) - V^\ell dp^\ell$$

$$-\frac{\Delta H\, dT}{T} = (V^s - V^\ell)\, dp^\ell + V^s d(\Delta p)$$

$$\Delta H = T\Delta S \qquad \Delta S = \frac{\Delta H}{T}$$

$$\approx V^s d(\Delta p)$$

$$\frac{dT}{T} = \frac{-V^s}{\Delta H} d(\Delta p) \quad \Big/ \int$$

$$\ln T = -\frac{V^s}{\Delta H}\Delta p + C = -\frac{V^s}{\Delta H}\frac{2\gamma}{r} + C$$

$$= -\frac{V^s}{\Delta H}\frac{2\gamma}{r} + \ln T_0$$

$$\ln T - \ln T_0 = \ln\frac{T}{T_0} = -\frac{V^s}{\Delta H}\frac{2\gamma}{r}$$ Gibbs-Thomson Eq

$$r_m = -\frac{V^s}{\Delta H}\frac{2\gamma}{\ln\frac{T_m}{T_0}}$$

How Do we measure Heat Capacity?

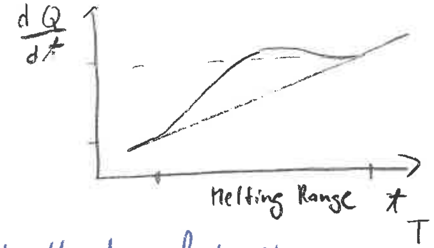
$$\frac{dQ}{dT} = \frac{dQ/dt}{dT/dt}$$

How do we measure Latent Heat?

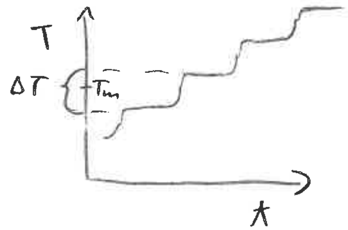
$\frac{dT}{dt}$ = constant

$$\Delta Q = \int \frac{dQ}{dt}\, dt = \Delta Q_c + \Delta H$$

$$= \int \left(\frac{\partial Q}{\partial t}\right)_c dt + \Delta H$$



How do we measure Latent Heat of Melting at a particular Temperature Range?



$$\Delta Q = \left(\frac{\partial Q}{\partial T}\right)_c \Delta T + \Delta H$$

$$\Delta H(T_m) = \Delta Q(T_m) - \left[\left(\frac{\partial Q}{\partial T}\right)_c (T_m)\right] \Delta T$$

$$m_w(T_m) = \frac{\Delta H(T_m)}{\varepsilon_w} \approx \frac{\Delta H(T_m)}{333\ J/g}$$

Pulp Beating

Specific Energy Consumption

$$SEC = \frac{\text{Energy}}{\text{mass}} = \frac{\text{Power} \cdot \text{time}}{\text{mass flow rate} \cdot \text{time}} = \frac{\text{Power}}{\text{mass flow rate}}$$

Is there a tare power?

$\Rightarrow$ Net power = Gross power − Tare power

?

Is SEC enough to characterize beating?

How about Intensity?

$$I = \frac{\text{Energy}}{\text{Number of Impacts}} = \frac{\text{Work}}{n_1 n_2 \frac{\omega}{2\pi} t} = \frac{\text{Power}}{n_1 n_2 \frac{\omega}{2\pi}}$$

Does it matter how long of an edge delivers any Energy Impact?

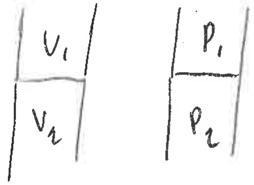
$$\text{Specific Edge Load} \equiv \frac{\text{Energy}}{\text{Number} \cdot \text{Edge length}} = \frac{\text{Power}}{n_1 n_2 \frac{\omega}{2\pi} L}$$

Does Beating Consistency have any effect on the Result?

Gravity Drainage Experiment

$$dV_1 = -dV_2$$

$$\Delta P = P_2 - P_1$$



$$\frac{dV_1}{dt} \propto \Delta P \qquad \text{Darcy's Law}$$

$$\frac{dV_1}{dt} = \frac{\Delta P \, A}{\eta R}$$

$R \equiv$ Filtration Resistance

$b \equiv$ thickness

$R \propto b \Rightarrow R = SFR_b \cdot b$

Specific Filtration Resistance:

$R \propto Bw \Rightarrow R = SFR_{Bw} \cdot Bw$

$$\frac{dV_1}{dt} = \frac{\Delta P}{b} \frac{A}{\eta \, SFR_b}$$

towards continuum:

$$\frac{dV_2}{A\,dt} = -\frac{dp}{dx} \frac{1}{\eta \, SFR_b}$$

Can we generalize this?

1-d Fick's Law

$$\frac{\partial^2 Q}{\partial t \, \partial A_\perp} = -D \frac{\partial^2 Q}{\partial x \, \partial V}$$

3-d Fick's Law

$$J_i \equiv \frac{\partial^2 Q}{\partial t \, \partial A_{\perp i}}$$

$$\phi \equiv \frac{\partial Q}{\partial V}$$

$$\bar{J} = -[D] \nabla \phi = -[D] \hat{e}_i \frac{\partial}{\partial x_i} \phi$$

$$\begin{pmatrix} J_1 \hat{e}_1 \\ J_2 \hat{e}_2 \\ J_3 \hat{e}_3 \end{pmatrix} = \begin{pmatrix} \bar{J}_1 \\ \bar{J}_2 \\ \bar{J}_3 \end{pmatrix} = -\begin{pmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{pmatrix} \begin{pmatrix} \hat{e}_1 \frac{\partial \phi}{\partial x_1} \\ \hat{e}_2 \frac{\partial \phi}{\partial x_2} \\ \hat{e}_3 \frac{\partial \phi}{\partial x_3} \end{pmatrix}$$

Flux Equation $\frac{\partial^2 Q}{\partial t \, \partial A_{\perp i}} = -D_i \frac{\partial^2 Q}{\partial x_i \, \partial V} = -D_i \frac{\partial \phi}{\partial x_i}$

Conductivity Equation $\frac{\partial^2 Q}{\partial t \, \partial A_i} = -k_i \frac{\partial T}{\partial x_i}$

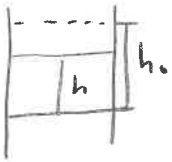
Using the Divergence Theorem

→ Diffusion Eq. $\frac{d\phi}{dt} = [D] \nabla^2 \phi$

Continuity Eq. $\frac{d\phi}{dt} + \nabla \cdot J = 0$

$$\frac{dV_1}{dt} = \frac{\Delta p A}{\eta \, SFR \, B_w} = -\frac{\rho g h A}{\eta \, SFR \, B_w} = -\frac{\rho g V}{\eta \, SFR \, B_w}$$

$$\frac{d \, V/V_0}{dt} = -\frac{\rho g \, V/V_0}{\eta \, SFR \, B_w} = -\frac{\rho g \, V/V_0 \, A}{\eta \, SFR \, (V_0 - V) c}$$



$$B_w = \frac{(V_0 - V) c}{A}$$

$$\frac{dt}{d \, V/V_0} = -\frac{\eta \, SFR \, (V_0 - V) c}{\rho g \, V/V_0 \, A} = -\frac{\eta \, SFR \, (V_0/V - 1) c}{\rho g \, A/V_0} = \left(1 - \frac{1}{V/V_0}\right) \frac{\eta \, SFR \, c h_0}{\rho g}$$

$$t(V/V_0 : 1 \to a): \quad t_a = \int_1^a \left(1 - \frac{1}{V/V_0}\right) \frac{\eta \, SFR \, c h_0}{\rho g} \, d \, V/V_0$$

$$= \frac{\eta \, SFR \, c h_0}{\rho g} \Big|_1^a \frac{V}{V_0} - \ln V/V_0 = \frac{\eta \, SFR \, c h_0}{\rho g} (a - 1 - \ln a)$$

$$a - \ln a - 1 + \ln 1$$

$$SFR = \frac{t_a \rho g}{\eta c h_0 (a - 1 - \ln a)}$$

$$\left[\frac{s \frac{kg}{\cancel{m^3}} \frac{\cancel{m}}{s^2}}{\cancel{\frac{kg\,m}{s^2}} \frac{s}{m^2} \frac{kg}{\cancel{m^3}} m} \right] = \left[\frac{m}{kg} \right]$$

$$\eta \approx 1{,}0 \cdot 10^{-3} \frac{Ns}{m^2}$$

Constant Pressure Difference

$$\frac{dV_2}{dt} = \frac{-\Delta P A}{\eta \, SFR \, B_w}$$

$$B_w = \frac{V_2 c}{A}$$

$$V_2 = \frac{B_w A}{c}$$

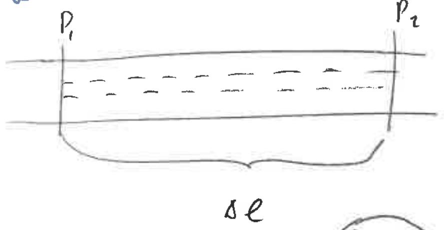
$$\frac{dB_w}{dt} = \frac{-\Delta P c}{\eta \, SFR \, B_w}$$

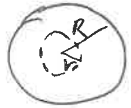
$$\frac{dt}{dB_w} = -\frac{\eta \, SFR \, B_w}{\Delta P c}$$

$$t_{B_{wf}} = \int_0^{B_{wf}} \frac{dt}{dB_w} \, dB_w = \int_0^{B_{wf}} -\frac{\eta \, SFR \, B_w}{\Delta P c} \, dB_w = -\frac{\eta \, SFR}{\Delta p c} \Big|_0^{B_{wf}} \frac{1}{2} (B_w)^2$$

$$= -\frac{\eta \, SFR \, (B_{wf})^2}{2 \Delta p c}$$

Hagen-Poiseuille Law





$$(P_2-P_1)\pi r^2 + \sigma_{lr}\, 2\pi r\, \Delta l = 0$$

$$(P_2-P_1)\pi r^2 + \eta\, \dot{\varepsilon}_{lr}\, 2\pi r\, \Delta l = 0$$

$$\dot{\varepsilon}_{lr} = \frac{d}{dt}\frac{du}{dr} = \frac{d}{dr}\frac{du}{dt} = \frac{d}{dr} v$$

$$\dot{\varepsilon}_{lr} = \frac{(P_2-P_1)\, r}{2\eta\, \Delta l} = \frac{dP}{dl}\frac{r}{2\eta}$$

$$\frac{dv}{dr} = \frac{dP}{dl}\frac{r}{2\eta}$$

$$dv = \frac{dP}{dl}\frac{r}{2\eta}\, dr$$

$$v = \frac{dP}{dl}\frac{r^2}{4\eta} + C$$

$$v(r=R) = 0 \Rightarrow C = -\frac{dP}{dl}\frac{R^2}{4\eta}$$

$$v(r) = \frac{dP}{dl}\frac{r^2-R^2}{4\eta}$$

$$\frac{dV}{dt} = \int_0^R 2\pi r\, v(r)\, dr = -\frac{dP}{dl}\frac{2\pi}{4\eta}\int_0^R rR^2 - r^3\, dr = -\frac{dP}{dl}\frac{2\pi}{4\eta}\Big|_0^R \frac{1}{2}r^2R^2 - \frac{1}{4}r^4$$

$$= -\frac{dP}{dl}\frac{2\pi}{4\eta}\frac{R^4}{4} = -\frac{dP}{dl}\frac{\pi R^4}{8\eta}$$

$$\frac{dV}{A_\perp dt} = \frac{1}{\pi R^2}\frac{\partial V}{\partial t} = -\frac{dP}{dl}\frac{R^2}{8\eta}$$

Rewrite Darcy's Law

$$\frac{\partial^2 V}{\partial t\, \partial A_\perp} = -\frac{K}{\eta}\frac{dP}{dl}$$

$K \equiv$ Permeability

$$K = \frac{1}{SFR_b} = \frac{1}{SFR_{Bw}\frac{Bw}{b}} = \frac{1}{SFR_{Bw}\, S}$$

Porosity Effect

$$\frac{\partial}{\partial A_\perp}\frac{\partial V}{\partial t} = \frac{\partial N}{\partial A_\perp}\frac{\partial}{\partial N}\frac{\partial V}{\partial t} = \frac{\mathbb{P}}{\pi R^2}\left[-\frac{\pi R^4}{8\eta}\frac{dP}{dl}\right] = -\mathbb{P}\frac{R^2}{8}\frac{1}{\eta}\frac{dP}{dl}$$

$$\Rightarrow K = \mathbb{P}\frac{R^2}{8}$$

$\mathbb{P} \equiv$ Porosity

Variable Size of Pores

$$K = \int \mathbb{P}\frac{R^2}{8}\, p(R)\, dR$$

Variable Geometry, Alignment

$$K \propto \mathbb{P} R_{ch}^2$$

$$SFR_b \propto \frac{1}{\mathbb{P} R_{ch}^2}$$

$$SFR_{Bw} \propto \frac{1}{S\, \mathbb{P} R_{ch}^2}$$

$$t_{Bwf} = \frac{\eta\, SFR_{Bw}\, (Bwf)^2}{2\Delta p\, c}$$

$$\propto \frac{\eta\, (Bw)^2}{\Delta p\, c\, S\, \mathbb{P} R_{ch}^2}$$

A Porous System Consisting of Parallel Tubes

$$\frac{dV}{dt} = \left(\frac{dV}{dt}\right)_1 + \left(\frac{dV}{dt}\right)_2 + \dots + \left(\frac{dV}{dt}\right)_n$$

$$\frac{dV}{A\,dt} = \frac{1}{A}\left[\left(\frac{dV}{dt}\right)_1 + \left(\frac{dV}{dt}\right)_2 + \dots + \left(\frac{dV}{dt}\right)_n\right] = \frac{1}{A}\left[A_1\left(\frac{dV}{A_1\,dt}\right)_1 + \dots\right]$$

If all tubes are of the same size

$$\frac{dV}{A\,dt} = \frac{nA_1}{A}\left(\frac{dV}{A_1\,dt}\right)_1 = -\mathbb{P}\,\frac{R^2}{8\eta}\,\frac{dp}{dl}$$

Tubes of not the same size

$$\frac{dV}{A\,dt} = -\frac{1}{A}\,\frac{dp}{dl}\,\frac{1}{8\eta}\left[A_1R_1^2 + A_2R_2^2 + \dots + A_nR_n^2\right]$$

$$= -\frac{A_p}{A}\,\frac{dp}{dl}\,\frac{1}{8\eta}\,\frac{A_1R_1^2 + A_2R_2^2 + \dots + A_nR_n^2}{A_p}$$

$$\frac{dV}{A\,dt} = -\mathbb{P}\,\frac{dp}{dl}\,\frac{1}{8\eta}\int P(A)\,R^2\,dA$$

$$= -\mathbb{P}\,\frac{dp}{dl}\,\frac{1}{8\eta}\int P(A)\,R^2\,2\pi R\,dR$$

$$= -\mathbb{P}\,\frac{dp}{dl}\,\frac{1}{8\eta}\int P(R)\,R^2\,dR$$

$$\int P(A)\,dA = 1$$

$$\int P(A)\,d(\pi R^2) = 1$$

$$\frac{d\pi R^2}{dR} = 2\pi R$$

$$d\pi R^2 = 2\pi R\,dR$$

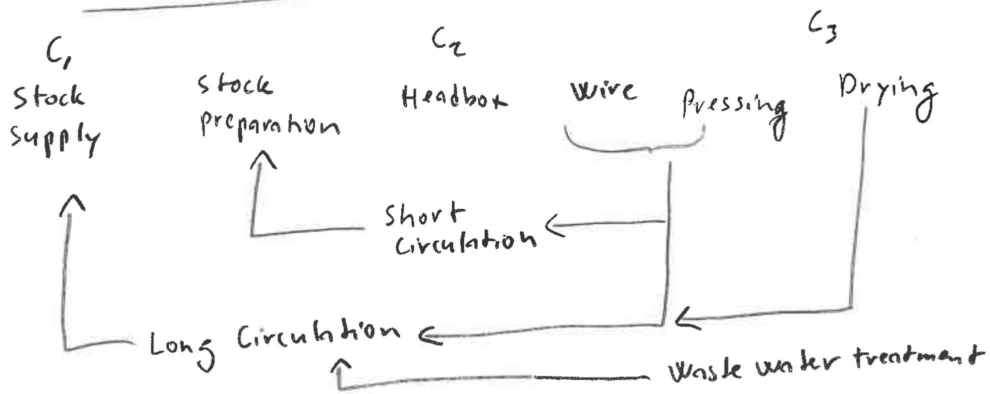
$$\int P(A)\,2\pi R\,dR = 1$$

$$P(R) = P(A)\,2\pi R$$

The Retention Problem

Particles pass through the wire
– in particular small particles!

- Retention chemicals
- White Water Recirculation



$$C_3 > C_1 > C_2$$

HC 0,2 ... 0,5

MC ~ 0,1

$$c = \frac{m_f}{m_f + m_w} = \frac{1}{1 + \frac{m_w}{m_f}}$$

$$m_w = m_f\left(\frac{1}{c} - 1\right)$$

Example values

Mass of water collected at the wire section

$$m_{w2} - m_{w3} = m_f\left(\frac{1}{C_2} - \frac{1}{C_3}\right) \approx m_f\left(\frac{1}{0{,}006} - \frac{1}{0{,}2}\right) \approx 162 \cdot m_f$$

Mass of water "collected" in stock preparation

$$m_{w1} - m_{w2} = m_f\left(\frac{1}{C_1} - \frac{1}{C_2}\right) \approx m_f\left(\frac{1}{0{,}1} - \frac{1}{0{,}006}\right) \approx -157 \cdot m_f$$

$\Rightarrow$ $157 \cdot m_f$ goes to short circulation
$5 \cdot m_f$ goes to long circulation
+ additional waters, from waste w. treatment, etc.

Is a fully closed water circulation possible?

Forming Anisotropy

What is the Effect of velocity difference between Jet and wire on the Fiber orientation Distribution?

Does this change from the top to the bottom?

Drying Shrinkage

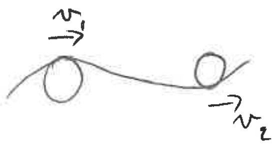
What is the effect of Drying Shrinkage on Properties?

- Stiffness
- Strength
- Ductility
- Hygroexpansivity

How can Drying Shrinkage be Controlled?

How do we Induce Wet Strain?

How can shrinkage be controlled in CD?



$$v_1 = \left(\frac{dx}{dt}\right)_1 = \frac{\Delta x}{\Delta t}$$

$$v_2 = \left(\frac{dx}{dt}\right)_2 = \frac{\Delta x + \Delta y}{\Delta t}$$

$$v_2 - v_1 = \frac{\Delta y}{\Delta t}$$

$$\varepsilon = \frac{\Delta y}{\Delta x} = \frac{\Delta t}{\Delta x}\frac{\Delta y}{\Delta t} = \frac{v_2 - v_1}{v_1} = \frac{v_2}{v_1} - 1$$

Wet Pressing and Density Effects

Spring Eq. $F = k\delta$ $\qquad V = Al = bwl$

Hooke's Law $\sigma = E\varepsilon$ $\qquad \frac{F}{A} = E\frac{\delta}{l}$

$$E = \frac{Fl}{A\delta} = \frac{F}{bw\varepsilon}$$

A/ <u>Material Properties (E, ρ) constant:</u>

$$F \propto A \propto bw$$

$$\Rightarrow k \propto bw$$

B/ <u>Basis weight constant</u> $\quad Bw = \frac{m}{A} = \frac{m}{lw} = \frac{m}{V}b = \rho b$

$$\Rightarrow \rho \propto b^{-1}$$

Fiber-Fiber Interaction Increases w. Increased Density

$$F \propto \rho^{\mu} \qquad \mu > 0$$

$$E = \frac{F}{bw\varepsilon} \propto F\rho \propto \rho^{1+\mu}$$

$$\sigma_c = \varepsilon_c^e E \qquad \varepsilon_c^e \propto \rho^{\gamma}$$

$$\sigma_c \propto \rho^{1+\mu+\gamma}$$

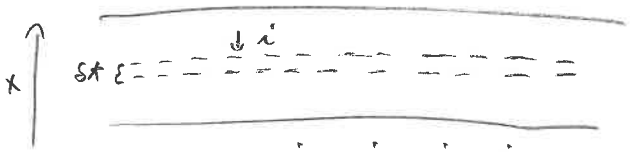
$E \propto (\rho - \rho_0)$?

How much is γ?

$\gamma > 0$?

$\gamma \approx 0$?

Kubelka - Munk Optics

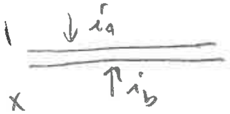


$$\frac{d i_R / i}{dx} \equiv S$$

$$i = i_R + i_T + i_A$$

$$1 = \frac{i_R}{i} + \frac{i_T}{i} + \frac{i_A}{i}$$

$$\frac{d i_A / i}{dx} \equiv K$$



$$d i_a = (s+k) i_a \, dx - s \, i_b \, dx \quad \Big| \cdot \frac{-1}{i_a}$$

$$d i_b = -(s+k) i_b \, dx + s \, i_a \, dx \quad \Big| \cdot \frac{1}{i_b}$$

$$\frac{d i_b}{i_b} - \frac{d i_a}{i_a} = -2(s+k)\,dx + s\left(\frac{i_b}{i_a} + \frac{i_a}{i_b}\right)dx$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{dx}{x} = d \ln x$$

$$d \ln i_b - d \ln i_a =$$

$$d(\ln i_b - \ln i_a) = d \ln \frac{i_b}{i_a}$$

$$\boxed{\frac{i_a}{i_b} \equiv r}$$

$$= -d \ln \frac{i_a}{i_b} = -d \ln r$$

$$-d \ln r = -2(s+k)\,dx + s\left(\frac{1}{r} + r\right)dx = -\frac{dr}{r}$$

$$-dr = -2(s+k)\, r \, dx + s(1+r^2)\,dx = s\,dx\left(r^2 - 2\,\frac{s+k}{s}\, r + 1\right)$$

$$-s\,dx = \frac{dr}{r^2 - 2ar + 1} \qquad a \equiv \frac{s+k}{s}$$

← Partial Fractions Decomposition

$$-2s\sqrt{a^2-1}\,dx = dr\left[\frac{1}{r-a-\sqrt{a^2-1}} - \frac{1}{r-a+\sqrt{a^2-1}}\right]$$

Partial Fractions Decomposition

$$f(x) = \frac{g(x)}{h(x)}$$

The Ratio changes its values most rapidly in the vicinity of $x = \alpha$, where $h(\alpha) = 0$.

$$\Rightarrow f(x) = \frac{A_1}{(x-\alpha_1)^{n_1}} + \frac{A_2}{(x-\alpha_2)^{n_2}} + \dots$$

1° How many terms should the Decomposition have?

- as many as distinct roots for $h(x) = 0$

2° What should be the exponents $n_1, n_2, \dots$?

- The degree of $h(x) = m \Rightarrow$

$$h(x) = A(x-\alpha_1)^{n_1}(x-\alpha_2)^{n_2}\dots(x-\alpha_k)^{n_k}$$

$$m = n_1 + n_2 + \dots + n_k$$

Example:

$$\frac{1}{r^2 - 2ar + 1} = \frac{1}{h(r)}$$

$$h(\alpha) = 0 \Rightarrow \alpha = a \pm \sqrt{a^2-1}$$

$$\frac{1}{h(r)} = \frac{1}{(r-\alpha_1)(r-\alpha_2)} = \frac{A_1}{r-\alpha_1} + \frac{A_2}{r-\alpha_2} = \frac{A_1}{r-a-\sqrt{a^2-1}} + \frac{A_2}{r-a+\sqrt{a^2-1}}$$

$$1 = A_1(r-\alpha_2) + A_2(r-\alpha_1) = A_1\left(r-a+\sqrt{a^2-1}\right) + A_2\left(r-a-\sqrt{a^2-1}\right)$$

$$= (A_1+A_2)(r-a) + (A_1-A_2)\sqrt{a^2-1}$$

$1 =$ constant for any $r \Rightarrow A_1 + A_2 = 0 \Rightarrow A_1 = -A_2$

$$\Rightarrow 1 = 2A_1\sqrt{a^2-1} \Rightarrow A_1 = \frac{1}{2\sqrt{a^2-1}}$$

$$A_2 = -\frac{1}{2\sqrt{a^2-1}}$$

(24)

$$\int_0^t 2s\sqrt{a^2-1}\,dx = \int_{R_0}^{R_t}\left[\frac{1}{r-a+\sqrt{a^2-1}} - \frac{1}{r-a-\sqrt{a^2-1}}\right]dr$$

$$2st\sqrt{a^2-1} = \Big/_{R_0}^{R_t} \ln\frac{r-a+\sqrt{a^2-1}}{r-a-\sqrt{a^2-1}} = \ln\frac{R_t-a+\sqrt{a^2-1}}{R_t-a-\sqrt{a^2-1}}\,\frac{R_0-a-\sqrt{a^2-1}}{R_0-a+\sqrt{a^2-1}}$$

$$r \equiv \frac{i_a}{i_b}$$

$$t\to\infty \Rightarrow R_0 \to R_\infty\,,\quad R_\infty-(a-\sqrt{a^2-1})\to 0$$

$$\Rightarrow R_\infty = a-\sqrt{a^2-1} = \frac{s+k}{s}-\sqrt{\left(\frac{s+k}{s}\right)^2-1} = 1+\frac{k}{s}-\sqrt{\left(1+\frac{k}{s}\right)^2-1}$$

$$= 1+\frac{k}{s}-\sqrt{\left(\frac{k}{s}\right)^2+2\frac{k}{s}}$$ Limit Reflectivity

$$R_\infty = a-\sqrt{a^2-1}$$

$$\frac{1}{R_\infty} = \frac{1}{a-\sqrt{a^2-1}} = \frac{a+\sqrt{a^2-1}}{\cancel{a^2}-\cancel{a^2}+1}$$

$$\frac{1}{R_\infty}+R_\infty = 2a$$

$$\frac{1}{R_\infty}-R_\infty = 2\sqrt{a^2-1}$$

Finite thickness w. non-reflecting Background $\Rightarrow R(t)=0$

$$2st\sqrt{a^2-1} = \ln\left(\frac{R_t-R_\infty}{R_t+R_\infty-2a}\,\frac{R_0+R_\infty-2a}{R_0-R_\infty}\right)$$

$$st\left(\frac{1}{R_\infty}-R_\infty\right) = \ln\frac{-R_\infty}{R_\infty-2a}\,\frac{R_0+R_\infty-2a}{R_0-R_\infty}$$

$$\exp\left[st\left(\frac{1}{R_\infty}-R_\infty\right)\right] = \frac{-R_\infty}{-\frac{1}{R_\infty}}\,\frac{R_0-\frac{1}{R_\infty}}{R_0-R_\infty} = R_\infty^2\,\frac{R_0-\frac{1}{R_\infty}}{R_0-R_\infty}$$

$$R_0-\frac{1}{R_\infty} = \left(\frac{R_0}{R_\infty^2}-\frac{1}{R_\infty}\right)\exp[\,]$$

$$R_0\left(1-\frac{\exp[\,]}{R_\infty^2}\right) = \frac{1}{R_\infty}(1-\exp[\,])$$

$$R_0 = R_\infty\,\frac{1-\exp[\,]}{R_\infty^2-\exp[\,]} = R_\infty\,\frac{\exp\left[st\left(\frac{1}{R_\infty}-R_\infty\right)\right]-1}{\exp\left[st\left(\frac{1}{R_\infty}-R_\infty\right)\right]-R_\infty^2}$$ Reflectivity

$$\text{Opacity} \equiv \frac{R_0}{R_\infty}$$