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ABSTRACT

A relationship between stress and deformation can be evaluated from the load and elongation recorded in the tensile test of a strip of paper. This relationship characterizes the fracture properties of the paper. It shows how the stress acting across a fracture region decreases as the deformation increases. The relationship can be used in strength analysis by the finite element method and for the determination of various conventional fracture parameters such as fracture energy. The theory of characterization, a test method, and test results for three papers are presented.

Application:

This method of characterization gives results that are suitable for strength analysis by the finite element method.

Characterization of tensile fracture properties of paper

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THE PRE-FRACTURE TENSILE properties of paper are usually characterized by its stiffness or by its stress–strain performance. The fracture properties of paper are characterized by its tensile strength and by some parameter that quantifies its toughness in fracturing. The tensile strength indicates the load-carrying capacity. The second parameter indicates the resistance to crack growth and, accordingly, the load-carrying capacity if the paper has a crack or a sharp notch.

A unified method for characterizing the tensile fracture properties has been developed. This characterization is made by an experimentally determined curve rather than by one or two parameters. As a result, the properties are defined more accurately. This method of characterization is also suitable for analyzing the strength of a paper product by the finite element method.

Our scope here is limited to the method of characterization, a test procedure, and test results for the case of uniaxial tension in either the machine direction or the cross-machine direction. [For the analysis of applied strength in biaxial, in-plane loading, a corresponding biaxial orthotropic model has been developed (*1*).]

BACKGROUND

The characterization of tensile fracture properties is related to cohesive crack modeling (*2–4*). In recent years, with the help of the finite element method, cohesive crack modeling has been successfully used to analyze the strength and fracture of products made of brittle and quasi-brittle materials. These kinds of materials include cement-based materials (*4–6*), fiber-reinforced plastics and laminates (*7, 8*), wood (*4, 9*), and adhesive joints (*10*). Many of these applications have their backgrounds in a model introduced in the 1970s (*2*) for fractures in concrete.

The cohesive crack type of modeling should also be useful for paper. The size of the fracture process zone is normally not small compared to the size of the defect or crack that initiates the fracture. Furthermore, the point of fracture initiation is not always a sharp crack or notch.

Our model is developed from a description of the performance of a strip of paper when it is loaded under tension. The specimen's instability under strain and the localization of strain during fracturing are taken into account.

TENSILE TEST PERFORMANCE

For the uniaxial, in-plane loading of a uniform paper strip, the experimentally observed failure mechanism is related to the length of the tested strip (*1, 11–15*). If the strip is long, the paper will rupture in a sudden and unstable manner at peak load. If the strip is short, the paper will exhibit a partly or completely stable performance with a gradual decrease in stress as the paper strip is elongated. These observations rely on the use of a machine in which the displacement can be controlled. (In load-control testing, by contrast, both long and short specimens are unstable.)

The top frame in **Fig. 1** depicts a short paper strip tested for tension. The corresponding graph shows the recorded stress, σ, vs. elongation, δ. It is assumed that the paper strip is short enough to make the performance stable, making it possible to record the entire curve for σ vs. δ. The condition for stable test performance can be defined in terms of the properties of the material and the length of the strip.

As long as the load, and accordingly the stress, is increasing, the deformation of the paper is approximately uniform along the strip. This deformation of the material is quantified by strain ε, where $\varepsilon = \delta/L$, with L being the initial span length of the paper strip.

After the instant when the peak load is reached, the deformation is no longer uniform along the strip. Instead, the subsequent damage and fracture of the material becomes localized to a narrow band, as depicted in the middle frame, Fig. 1B. Eventually a fracture section forms that separates the strip into two parts.

As the elongation of the strip continues, the deformation and damage in the fracture region increase, and the stress gradually decreases. The performance of the material in the fracture region is completely different from the performance outside the fracture region. Outside the fracture region, the paper is experiencing "unloading," meaning that the strain is decreasing as the load is decreasing.

Before peak stress, the elongation is equal to the strain times the initial span length:

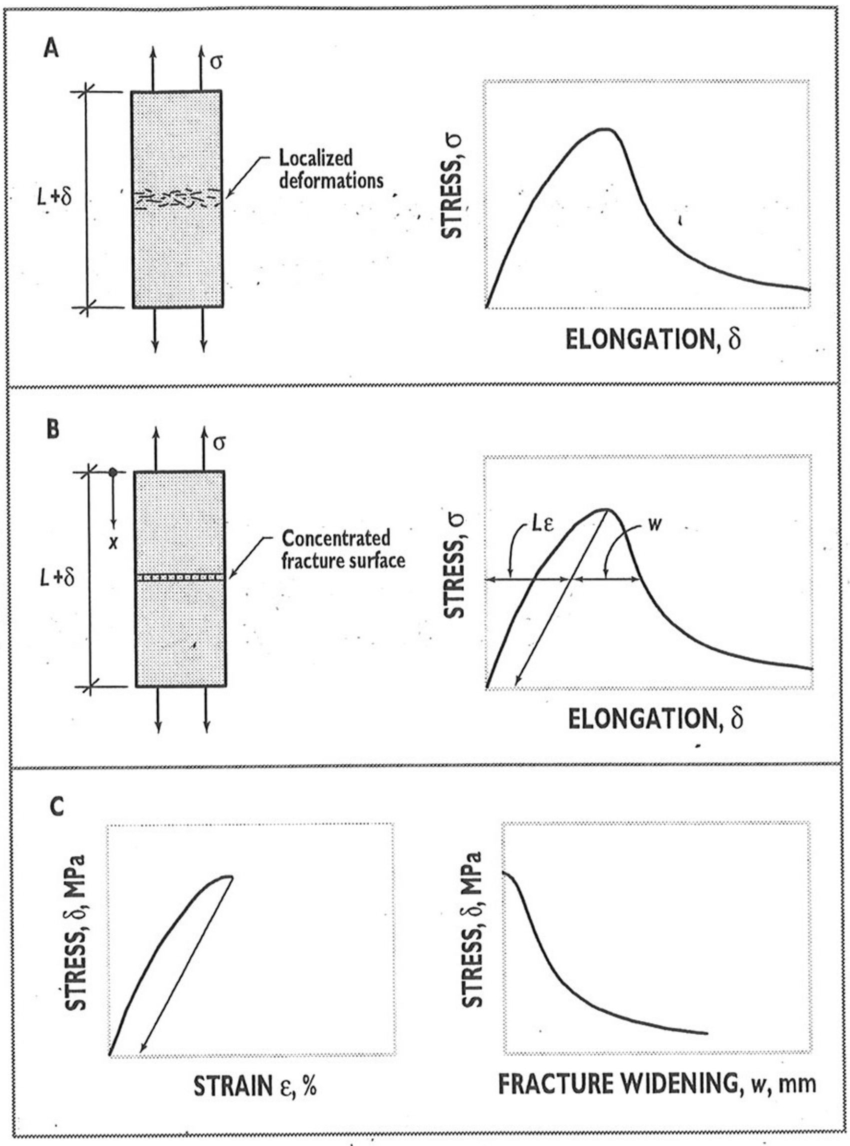
$$\delta = \varepsilon \times L \quad (1)$$

After peak stress, the total elongation of the strip is increased by fracture-induced deformation:

$$\delta = \varepsilon L + w \quad (2)$$

where w is the additional fracture-induced deformation that develops in the localized damage band or fracture section:

$$w(\sigma) = \delta(\sigma) - \varepsilon(\sigma)_{unl} L \quad (3)$$



1. Tensile fracture performance (top), model of the tensile fracture performance (middle), and characterization of the tensile properties of the material (bottom).

where $\varepsilon(\sigma)_{unl}$ represents the stress vs. strain performance of the material during unloading from peak stress.

For characterizing the performance of the paper, two curves are needed. One conventional $\sigma(\varepsilon)$ curve defines the performance of the material outside the fracture region during loading and unloading, and one $\sigma(w)$ curve defines the performance of the material in the fracture region. These curves are shown in the left and right graphs, respectively, in Fig. 1C.

Knowing the unloading performance of the material from the current test or from previous tests, we obtain the stress-widening $\sigma(w)$ curve from the recorded $\delta(\sigma)$ curve by subtracting the unloading deformations, as indicated in Fig. 1B. The result is shown in Fig. 1C. If the unloading performance of the paper is not known in advance, it may be necessary to record not only σ and δ but also the elongation along some part of the strip that does not include the fracture region.

Tests of paperboard have shown that the stiffness during reloading, after unloading from peak load, is approximately equal to the initial elastic stiffness (*1*), as illustrated in Fig. 1. This result means that the damage during loading up to the peak stress is negligible, with only plastic deformation taking place (*16*).

PARAMETERS DERIVED FROM THE σ(*w*) CURVE

The $\sigma(w)$ curve characterizes the tensile fracture properties of the paper. These properties may be affected by the rate of deformation, moisture content, and temperature. Moreover, since paper is anisotropic, the properties can be expected to be different for different orientations of the load. The $\sigma(w)$ curve can also be used for a direct implementation in finite element fracture analysis (*1*). From the curve, the tensile strength σ_f and the fracture energy G_f may also be evaluated:

$$\sigma_f = \sigma(0) \quad (4)$$

$$G_f = \int_0^{w_0} \sigma \, dw \quad (5)$$

where w_0 is the widening of the fracture zone that corresponds to complete separation of the material. The steepest slope of the $\sigma(w)$ curve is indicated by a parameter denoted by N:

$$N = (-d\sigma/dw)_{max} \quad (6)$$

The fracture energy G_f can be compared directly with fracture energy parameters used in other approaches to fracture mechanics: the critical energy release G_c in linear fracture mechanics and the critical J-integral value J_c in J-integral nonlinear fracture mechanics. If the size of the fracture process region is small compared to the crack and ligament length, one may expect G_f, G_c, and J_c to coincide (*17*). Moreover, G_f is directly related to the concept of the essential work of fracture. Material property testing applications of linear elastic fracture mechanics (*18*), the J-integral (*19, 20*), and the essential work of fracture in paper (*21*) may be found in the literature. Knowing G_f or G_c, we can calculate the fracture toughness K_c also, provided the elastic stiffness parameters of the material are known.

As mentioned, the parameter N represents the steepest slope of the curve. N is useful when conditions of stability are being formulated. N is also of interest in general estimations of paper properties, and it may be more relevant than the fracture energy G_f (*1*). G_f represents the total fracture energy, including the work corresponding to the commonly long tail of the $\sigma(w)$ curve. The stress transfer corresponding to this tail is activated only at a late stage of fracture—at such a late stage in many cases that it gives no beneficial effect (*1*).

In fracture mechanics analysis, it is often convenient to normalize the size of the structure, product, or paper to an intrinsic length of the material. The ratio $(K_c/\sigma_f)^2$ is often used for this purpose. In the method of characterization discussed here, the corresponding intrinsic length of the material is defined by Eq. 7:

$$length_{ch} = E\, G_f/\sigma_f^2 \quad (7)$$

where E is the elastic stiffness of material in the direction of loading. The intrinsic, or the characteristic length, $length_{ch}$, may be regarded as a kind of measure of the toughness of the fracture process region at the tip of a growing crack. Its value corresponds to the size of the fracture process region. Values for $length_{ch}$ are on the order of 10,000 mm for steel-fiber-reinforced concrete, 100 mm for wood fiber board, about 30-60 mm for paper, and about 0.5 mm for glass (*4*).

TEST METHOD

In principle, a single tensile test is sufficient to determine the $\sigma(w)$ and $\sigma(\varepsilon)$ curves. However, because of the stability requirement for testing $\sigma(w)$, the paper strips need to be shorter than those used in conventional tensile tests. Therefore, separate tests should be made, one for $\sigma(w)$ and one for $\sigma(\varepsilon)$.

The $\sigma(\varepsilon)$ test

The evaluation of strain ε pertains to the performance up to peak stress. However, the small amount of deformation that occurs before peak stress makes it difficult to achieve good accuracy if the paper strip is as short as needed in the tests for $\sigma(w)$. Moreover, the size and geometry of the specimen affect the $\sigma(\varepsilon)$ performance somewhat (*10*). Therefore, an established method, say TAPPI T494 os-70, should be used for testing the $\sigma(\varepsilon)$ performance.

The $\sigma(w)$ test

For the $\sigma(w)$ tests, at least five nominally equal test specimens should be cut in each principal direction. The test specimens should be 15 mm wide and long enough to be clamped in jaws 5 mm apart. The rate of deformation of the 5 mm test span should be 0.25 mm/min. The experimental conditions should follow T494 os-70.

The tests are best carried out by a stiff universal testing machine with a stiff load cell. A test setup that is not stiff enough may result in unstable test performance. Preferably, the paper strips are held by pneumatic vice-grips between parallel jaw faces. This type of grip has the advantage of allowing accurate fixation without any sliding, as well as rapid and easy loading of the paper strips. For a strong paper with a basis weight greater than about 150 g/m², serrated jaw faces may be used to avoid sliding. Since there is a risk that serrated jaw faces may damage the paper, flat jaws treated with emery paper are recommended for papers of less than 150 g/m².

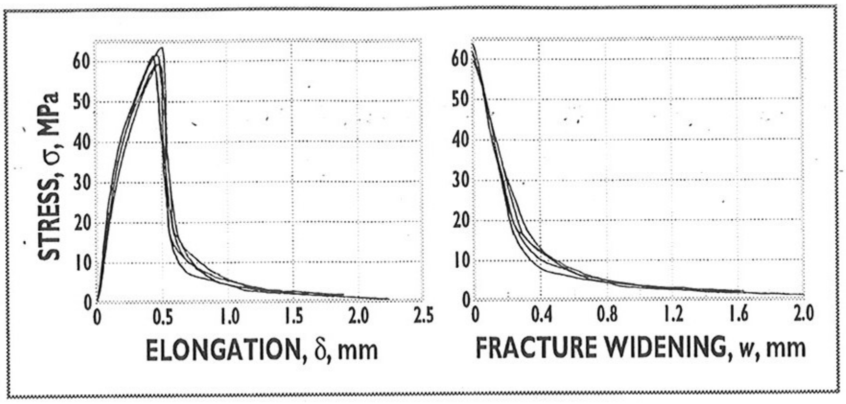
The elongation, δ, may be recorded as the movement of the crosshead. The movement of the crosshead includes, in addition to the elongation of the paper, possible deformation of the load cell, the grip, and other parts of the testing machine. However, as long as these undesired additional deformations are linear elastic, they will not affect the test result, since the elastic deformation is subtracted from the total deformations in the evaluation of w.

The recorded load-elongation curve is transformed into a $\sigma(w)$ curve by Eq. 8:

$$w(\sigma) = \delta(\sigma) - \delta(\sigma)_{unl} \quad (8)$$

	Basis wt., g/m²	Thick-ness, mm	Den-sity, kg/m³
Newsprint	45	0.072	643
Kraft paper	70	0.100	700
Paperboard	120	0.154	779

1. Papers tested



2. Performance of a 120 g/m² paperboard loaded in the machine direction: stress vs. elongation of paper strips (left) and corresponding stress vs. fracture widening curves (right).

Figure 2 shows representative test recordings and the corresponding $\sigma(w)$ curves obtained for a paperboard with a basis weight of 120 g/m².

Condition for stability

After peak load, elastic strain energy is released as the stress is decreasing. If this release of energy is greater than the energy required to develop the fracture in the fracture zone, the fracture development becomes unstable. The result is a test with a sudden drop in the load, which would be characteristic of a brittle specimen. Such a test performance must be avoided.

To analyze stability, we consider the change in load and the total deformation during a short increment of time. The total deformation is the sum of the deformation of the paper and the deformation of the testing machine:

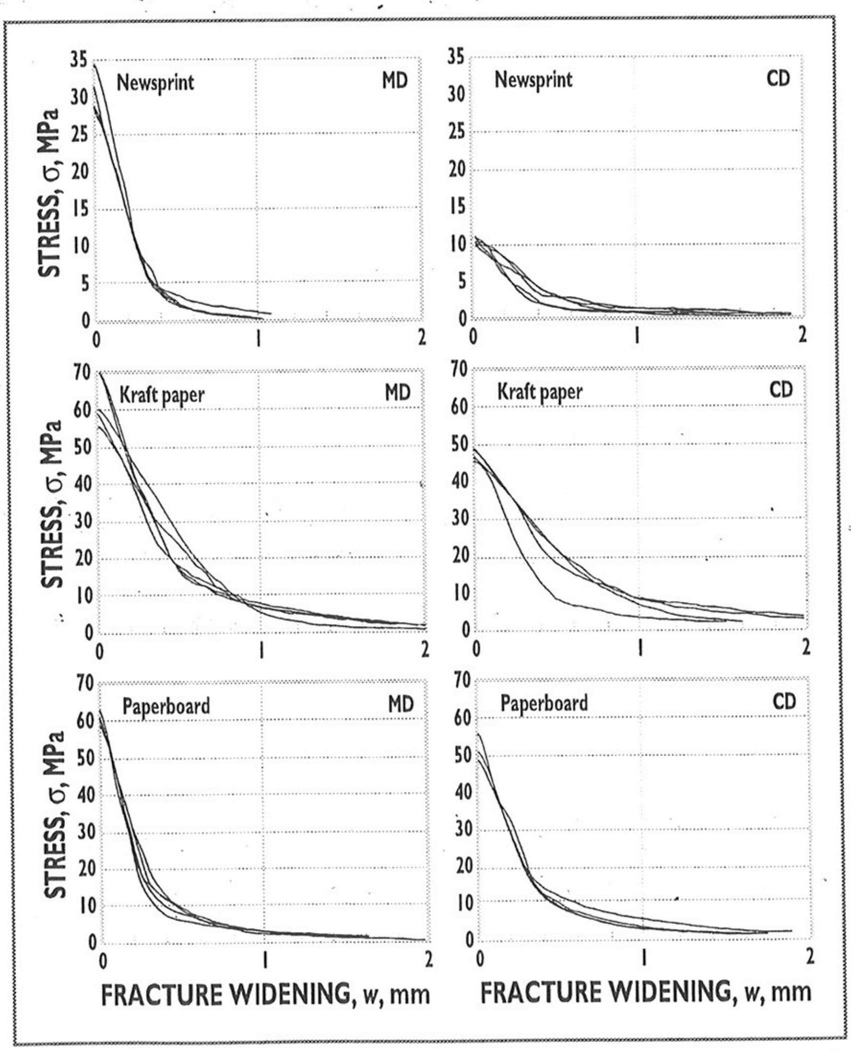
$$\delta = \sigma A/k + w + L\,\varepsilon \qquad (9)$$

where k is the stiffness of the testing machine and A is the specimen cross-section area. To avoid instability, a nonzero increment in total elongation is required to pursue the fracture and give a decrease in the load:

$$(\Delta\delta)/\Delta\sigma = \Delta(\sigma A/k)/\Delta\sigma + \Delta w/\Delta\sigma + \Delta(L\,\varepsilon)/\Delta\sigma < 0 \qquad (10)$$

Since $\Delta\varepsilon/\Delta\sigma = 1/E$ and $(-\Delta w/\Delta\sigma)_{\min} = 1/N$, this stability condition may be written as

$$L < E/N - (A\breve{E})/k \qquad (11)$$



3. Curves of σ(w) for a newsprint (45 g/m²), a kraft paper (70 g/m²), and a paperboard (120 g/m²) in machine and cross directions.

	Elastic stiffness, E		Strength, σ_f		Fracture energy, G_f		Steepest slope, N,
	MPa	Coeff.	MPa	Coeff.	J/m²	Coeff.	GPa/m
Machine direction							
Newsprint	4600	17%	32	11%	7100	12%	87
Kraft	3500	9%	57	5%	29000	2%	89
Paperboard	6800	2%	58	2%	17000	6%	154
Cross-machine direction							
Newsprint	870	2%	11	3%	4500	15%	24
Kraft	2500	4%	33	6%	24600	19%	57
Paperboard	3600	2%	37	4%	18500	8%	108

Coeff. = coefficient of variation.
Newsprint = 45 g/m². Kraft paper = 70 g/m². Paperboard = 120 g/m².

II. Test results for elastic stiffness, strength, fracture energy, and the steepest slope of the curve

	E/ρ, MPa/(kg/m³)	σ_f/ρ, kPa/(kg/m³)	G_f/ρ, J/m²/(kg/m³)	N/ρ, MPa/m/(kg/m³)	$Length_{ch}$, mm
Machine direction					
Newsprint	7.1	50	11	135	32
Kraft	5.0	82	41	127	31
Paperboard	8.7	74	22	197	35
Cross-machine direction					
Newsprint	1.3	17	7	37	34
Kraft	3.6	47	35	82	57
Paperboard	4.6	48	24	139	47

Newsprint = 45 g/m². Kraft paper = 70 g/m². Paperboard = 120 g/m².

III. Test results for specific elastic stiffness, specific strength, specific fracture energy, the specific slope for the steepest slope of the curve, and the characteristic length

Equation 11 shows that high stiffness of the testing machine facilitates stability and that the theoretical upper limit for the length of the paper strip is two times the characteristic length of the paper. This limit, $L < 2\ length_{ch}$, is obtained for a stiff testing machine, giving $AE/k = 0$ and a single straight line shape of the $\sigma(w)$ curve, giving $E/N = 2\ EG_f/\sigma_f^2 = 2\ length_{ch}$.

Since $d\sigma/dw$ varies during the course of fracture, the test performance may become partly stable even for paper strip lengths greater than as given in Eq. 11. The theoretical result, which indicates the effect of paper strip length on fracture performance, appears to agree with experimental observations reported elsewhere (*1*, *13*).

TESTS

Two groups of tests were carried out, one group to verify this method of characterization and one group to determine the $\sigma(w)$ curve for various papers in the machine and cross-machine directions. Here, we present results for the second group only.

In the first group, we tested one paper, which was a paperboard. We verified that the $\sigma(w)$ curve gives a true characterization of the fracture properties of paper, just as $\sigma(\varepsilon)$ characterizes the pre-fracture properties.

In the second group, we tested three papers—a newsprint, a kraft paper, and a paperboard. **Table I** gives the basis weights, thicknesses, and densities of the papers. These tests were tests of $\sigma(w)$ for various papers in the machine and cross-machine directions.

Figure 3 shows the $\sigma(w)$ curves obtained for newsprint, kraft paper, and paperboard in the machine and cross-machine directions. For each paper, the $\sigma(w)$ curve has a first part that is approximately linear, where the stress is decreasing comparatively rapidly as the fracture zone deformation is increasing. This first part extends to a fracture zone widening, w, equal to about 0.3–0.4 mm. The first part of the curve corresponds to the failure of fibers and fiber-to-fiber bonds.

After the first part is a tail, where the stress slowly approaches zero. In this second part, the separation of the material is complete when w has reached about 1–2 mm. This part of the curve probably corresponds to fiber-to-fiber friction when fibers are pulled out.

The shape of $\sigma(w)$ curve in the machine direction is similar to that in the cross-machine direction. However, the strength is greater in the machine direction, while the ductility, reflected in the smaller slope of the curve, is better in the cross direction. The difference between the MD profile and the CD profile is large for newspaper, but the differences are less pronounced for kraft paper and paperboard. For paperboard, the fracture energy was even found to be somewhat greater in the cross-machine direction than in the machine direction.

Table II indicates the fracture energy G_f for each paper, along with the coefficients of variation. Test results on modulus of elasticity E and tensile strength σ_f are also presented, along with their coefficients of variation. The tests of E and σ_f were carried out on paper strips with $L = 150$ mm and $B = 15$ mm. Results are also given for N, the steepest slope of the curve.

Table III gives the specific fracture energy, defined as G_f divided by the density of the paper, along with values for E/ρ, σ_f/ρ, and N/ρ. Table III also give results for the characteristic length, $length_{ch}$.

Kraft paper is seen to have the best specific fracture energy, probably reflecting the high demand for good mechanical properties for this kind of paper. As indicated by the coefficients of variation, the scatter in G_f is generally greater than in E and σ_f. On the other hand, the load-carrying capacity of a paper product is at most proportional to the square root of G_f. Therefore, it would be fair to half the coefficient of variation of G_f before comparing it with the coefficient of variation of σ_f. Although the papers tested are of different kinds, the characteristic lengths of the papers are about the same, at about 40 mm, both in the machine and cross directions (*1*).

SUMMARY

We propose a characterization of the tensile fracture properties of paper by a $\sigma(w)$ curve. The $\sigma(w)$ curve represents the stress σ acting across a fracturing section versus the fracture-induced widening of the section. In terms of a cohesive crack model, w is the distance between the two crack faces. From the $\sigma(w)$ curve, various material property parameters such as fracture energy, G_f, can be evaluated.

We propose a test method to determine $\sigma(w)$. Tests were carried out to verify the method and to determine the $\sigma(w)$ curves for a newsprint, a kraft paper, and a paperboard loaded in the machine and cross-machine directions.

Based on this method, several application analyses have been performed to predict the in-plane fracture of paper (*1*). These analyses show good agreement between experimental results and finite element simulations. ***TJ***

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