Measurement, Scaling, Instrumentation

27. 2. 2007

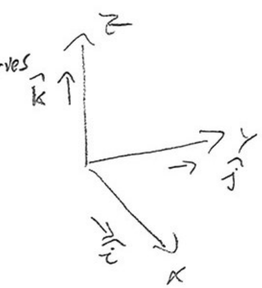
1

Measurement of Volume

1° Stereometric $\begin{cases} \text{Determine some boundaries} \\ \text{Adopt shape assumptions} \\ \text{Compute volume} \end{cases}$

2° Fluid Replacement

3° Buoyancy



Vectorial line elements

$d\bar{l} = \hat{l}\, dl$

$d\bar{i} = \hat{i}\, dx$

$d\bar{j} = \hat{j}\, dy$

$d\bar{k} = \hat{k}\, dz$

$$l = \int_A^B \hat{l}\cdot d\bar{l} = \int_A^B dl = \hat{l}\cdot\int_A^B d\bar{l} = \hat{l}\cdot\left[\int_A^C d\bar{i} + \int_C^B d\bar{j}\right]$$

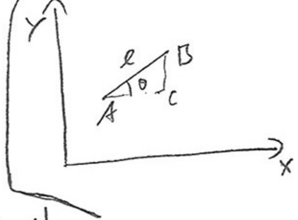
$$= \int_A^C \hat{l}\cdot d\bar{i} + \int_C^B \hat{l}\cdot d\bar{j}$$

$$= \int_A^C \cos\theta\, dx + \int_C^B \sin\theta\, dy$$

$$= |C-A|\cos\theta + |B-C|\sin\theta$$

$$= |C-A|\frac{|C-A|}{|B-A|} + |B-C|\frac{|B-C|}{|B-A|} = |B-A|$$

$$|B-A|^2 = |C-A|^2 + |B-C|^2$$



$$dl^2 = dx^2 + dy^2$$

$$dl = \sqrt{dx^2+dy^2} = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

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2

Surface Area Element

$$d\bar{S} = \hat{n}\, dS$$

Surface Area Element in xy-plane

$$d\bar{S}_{xy} = \hat{k}\, dS_{xy} = \hat{k}\,(\hat{k}\cdot d\bar{S}) = \hat{k}(\hat{k}\cdot\hat{n})dS = \hat{k}\cos\varphi\, dS$$

$$= \hat{k}\, dx\, dy = d\bar{i}\times d\bar{j} = \hat{i}\, dx \times \hat{j}\, dy$$

$$= \hat{k}\, dx\, dy$$

$$\bar{A}\times\bar{B} = A_i B_j \hat{e}_k \varepsilon_{ijk}$$

$$= \begin{bmatrix} e_i & e_j & e_k \\ A_i & A_j & A_k \\ B_i & B_j & B_k \end{bmatrix}$$

$$\hat{i}\times\hat{j} = \begin{bmatrix} \hat{e}_i & \hat{e}_j & \hat{e}_k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \hat{e}_k = \hat{k}$$

Area $A = \int \hat{n}\cdot d\bar{S} = \int dS$

In xy-plane

$$A_{xy} = \int \hat{k}\cdot d\bar{S}_{xy} = \int S_{xy} = \iint dx\, dy = \int \hat{k}\cdot\hat{n}\, dS = \int \cos\varphi\, dS$$

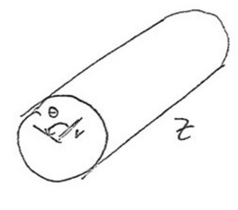
Volume

$$V = \int_V dV = \iiint dx\, dy\, dz = \int A_\perp(z)\, dz = \int A_{xy}(z)\, dz =$$

$$\int A(z)\cos\varphi\, dz$$

In Cylindrical co-ordinates

$$A = \int_0^{2\pi}\int_0^{r} r'\, dr'\, d\theta = \int_0^{2\pi} \tfrac{1}{2} r^2\, d\theta = \pi r^2$$



$$V = \int_0^l A(z)\, dz$$

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Let us compute the volume of a cylinder

$$V = \int_0^\ell A(z)\,dz = \int_0^\ell \pi r^2 dz = \pi r^2 \ell$$

And the volume of a cone : $r = az$ $\Rightarrow r_\ell = a\ell \Rightarrow a = \frac{r_\ell}{\ell}$

$$V = \int_0^\ell A(z)\,dz = \int_0^\ell \pi (az)^2 dz = \pi a^2 \Big|_0^\ell \frac{1}{3} z^3 = \frac{\pi a^2 \ell^3}{3} = \frac{\pi r_\ell^2 \ell}{3}$$

The volume of a frustum of a cone $r = r_0 + az$

$$V = \int_0^\ell A(z)\,dz = \int_0^\ell \pi (r_0 + az)^2 dz = \pi \int_0^\ell (r_0^2 + 2r_0 az + a^2 z^2)\,dz$$

$$= \pi \Big|_0^\ell r_0^2 z + r_0 a z^2 + \frac{1}{3} a^2 z^3 = \pi \left[r_0^2 \ell + r_0 a \ell^2 + \frac{1}{3} a^2 \ell^3 \right]$$

What is $r(\ell)$? $r(\ell) = r_0 + a\ell \Rightarrow a = \frac{r_\ell - r_0}{\ell}$

$$V = \pi \ell \left[r_0^2 + r_0 (r_\ell - r_0) + \frac{1}{3} (r_\ell - r_0)^2 \right]$$

$$= \frac{\pi \ell}{3} \left[r_\ell^2 + r_0 r_\ell + r_0^2 \right] = \frac{\pi \ell}{12} \left[d_0^2 + d_0 d_\ell + d_\ell^2 \right]$$

What if our object is not strictly a frustum of a cone ?

- Within any small section, shape difference will be negligible...
- The same applies even for a cylinder...

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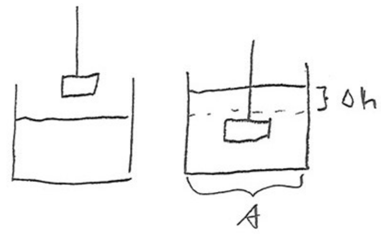
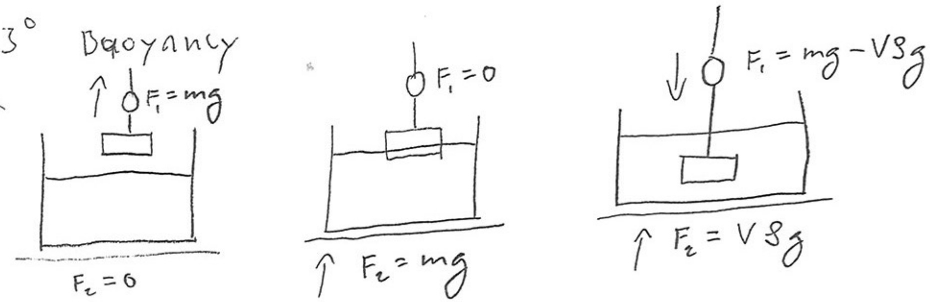
2° Fluid Replacement



$V = A \Delta h$

3° Buoyancy



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Measurement of mass

1° Spring displacement

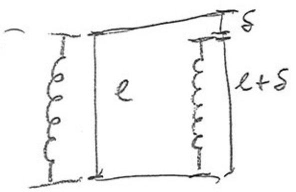
2° Balance of forces

3° Balance of torque moments

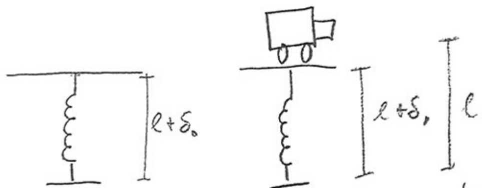
4° Buoyancy - fluid replacement

- others

1°



Spring Equation $F = k\delta$

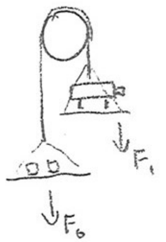


$F_0 = k\delta_0$

$F_1 = k\delta_1 = k(\delta_0 + \Delta\delta_1)$
$= F_0 + \Delta F_1$

$\Delta F_1 = k\,\Delta\delta_1$

2°

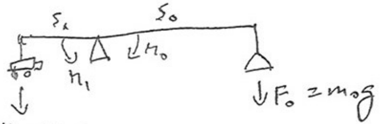


$F_1 = F_0$

$mg = m_0 g$

$m_1 = m_0$

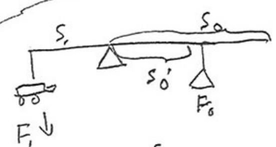
3°



$F_1 = m_1 g$

$M_1 = M_0$

$s_1 m_1 g = s_0 m_0 g \Rightarrow m_1 = \frac{s_0}{s_1} m_0$



$M_1 = s_1 m_1 g + \int_0^{s_1} \frac{dm}{ds} g\, s\, ds$

$M_0 = s_0' m_0 g + \int_0^{s_0} \frac{dm}{ds} g\, s\, ds$

$m_1 = \frac{s_0' m_0 + \int_0^{s_0} \frac{dm}{ds} s\, ds - \int_0^{s_1} \frac{dm}{ds} s\, ds}{s_1}$

4.3.2007

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Measurement of mass cntd

4°

$F_0 = \rho V g = \rho A h g$

$F_1 = m_1 g$

$F_0 + F_1 = 0 \Rightarrow F_1 = -F_0$

$m_1 = -\rho A h$

Moisture Content $\frac{m_w}{m_w + m_o}$

Moisture Ratio $\frac{m_w}{m_o}$

Dryness $\frac{m_o}{m_o + m_w}$

Fiber Saturation Point   Cell Wall Maximum $\left(\frac{m_w}{m_o}\right)$

Internal Energy Change

$$dU = TdS - pdV + \mu dN$$

$$U = TS - pV + \mu N \quad \text{Fundamental Eq.}$$

T Temperature
S Entropy
p Pressure
V Volume
μ Chemical potential
N Number of particles

Gibbs Free Energy

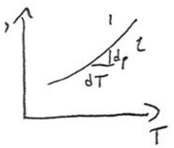
$$G = U - TS + pV = \mu N$$

Gibbs Free Energy Change

$$dG = dU - SdT - TdS + pdV + Vdp$$
$$= \mu dN - SdT + Vdp$$

Coexistence of two phases: $\boxed{\mu(p,T)}_1 = \boxed{\mu(p,T)}_2$

$$\Rightarrow \boxed{G = \mu N}_1 = \boxed{G = \mu N}_2$$



$$\Delta G_1 = -S_1 dT + V_1 dp$$
$$\Delta G_2 = -S_2 dT + V_2 dp$$

$\Delta G_1 = \Delta G_2 \Rightarrow$

$-S_1 dT + V_1 dp = -S_2 dT + V_2 dp$

$(S_2 - S_1) dT = (V_2 - V_1) dp$ $TS = Q$, Heat

$\frac{dp}{dT} = \frac{S_2 - S_1}{\ldots} = \frac{\Delta S}{\ldots} = \frac{\Delta Q}{T\Delta V} = \frac{\Delta H}{T\Delta V}$ Clausius-Clapeyron Eq

Clausius-Clapeyron Continued

$pV = nRT$

$\Delta V \approx V_{vapor}$ $V_{vapor} \approx \frac{nRT}{p}$

$$\frac{dp}{dT} = \frac{\Delta H}{T\Delta V}$$

$$\frac{dp}{dT} = \frac{\Delta H p}{nRT^2}$$

$$\frac{dp}{p} = \frac{\Delta H}{nRT^2} dT \Bigg| \int$$

$m = n\, m_{mol}$

$n = \frac{m}{m_{mol}}$

$$\ln p = \frac{-\Delta H}{nRT} + C = -\frac{\Delta H}{m}\frac{m_{mol}}{RT} + C$$

For Water:

$$p = e^{-\frac{\Delta H}{m}\frac{m_{mol}}{RT}} e^{C} = C_2\, e^{-\frac{\Delta H}{m}\frac{m_{mol}}{RT}}$$

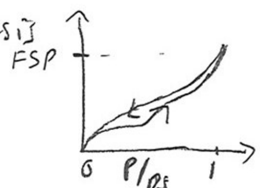
$\frac{\Delta H}{m} \approx 2260\ \mathrm{J/g}$

$m_{mol} = 18\ \mathrm{g/mol}$

$\equiv p_s$ SATURATION VAPOR PRESSURE $R = 8{,}31\ \frac{\mathrm{J}}{\mathrm{mol\,K}}$

Relative Vapor Pressure
Water Activity
Relative Humidity
$\Bigg\} \ p/p_s \Rightarrow$ Equilibrium moisture content

Hysteresis



Why does MC Increase w. p/p_s ?

KELVIN Eq II: Vapor pressure in droplet of radius r

$$p_r = p_\infty\, e^{\frac{V2\gamma}{RTr}} \Big| : p_s$$

γ = surface tension
V = molar volume

For largest droplet $1 = \frac{p_\infty}{p_s} e^{\frac{V2\gamma}{RTr}}$

$$\frac{p}{p_s} = e^{-\frac{V2\gamma}{RTr}}$$

Check the Eq. for $\begin{cases} p \to 0 \\ p \to p_s \end{cases}$

$$-\frac{V2\gamma}{RTr} = \ln p/p_s$$

$$r_{max} = -\frac{V2\gamma}{RT \ln p/p_s}$$

11. 3. 2007

9

What happens to water activity as a function of Temperature?

$$\frac{(P/p_s)_2}{(P/p_s)_1} = \frac{P/p_{s2}}{P/p_{s1}} = \frac{p_{s1}}{p_{s2}} = \frac{e^{-\frac{\Delta H}{m}\frac{m_{mol}}{RT_1}}}{e^{-\frac{\Delta H}{m}\frac{m_{mol}}{RT_2}}} = e^{\frac{\Delta H}{m}\frac{m_{mol}}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}$$

Say, $\begin{cases} T_1 = 293 \text{ K} \\ T_2 = 373 \text{ K} \end{cases} \Rightarrow \frac{1}{T_2} - \frac{1}{T_1} = \left(\frac{1}{373} - \frac{1}{293}\right)\frac{1}{K} \approx (268 \cdot 10^{-5} - 341 \cdot 10^{-5})\frac{1}{K}$

$$\approx -73 \cdot 10^{-5} \frac{1}{K}$$

$$\approx -\frac{1}{1366 \text{ K}}$$

$$\frac{(P/p_s)_2}{(P/p_s)_1} = e^{-2260 \, J/g \, \frac{18 \, g/mol}{8{,}31 \frac{J}{mol\,K} \, 1366 K}} \approx e^{-3{,}58} \approx 0{,}028$$

6. 3. 2007

10

Determination of FSP

Solute of molecules, concentration $c_1 = \frac{m}{V_1}$

porous substance added $c_2 = \frac{m}{V_2} = \frac{m}{V_1 - V_p}$

$$\frac{m}{c_2} = V_1 - V_p \Rightarrow V_p = V_1 - \frac{m}{c_2} = m\left[\frac{1}{c_1} - \frac{1}{c_2}\right]$$

$$FSP = \frac{V_p \rho_w}{m_0} = \frac{m_w}{m_0}$$

Moisture Content and Electrical Conductivity

Spring Eq.

$$\underset{[N]}{F} = \underset{\left[\frac{N}{m}\right]}{K} \, \underset{[m]}{\delta}$$

Hooke's Law

$$\underset{\left[\frac{N}{m^2}\right]}{\frac{F}{A}} = \underset{\left[\frac{N}{m^2}\right]}{E} \, \underset{[\,]}{\frac{\delta}{l}}$$

Potential difference Eq.

$$\Delta Q = R \, I$$

$[V]$ $\left[\frac{V}{A}\right]$ $[A]$

$\left[\frac{J}{C}\right]$ $\left[\frac{Js}{C^2}\right]$ $\left[\frac{C}{s}\right]$

$[\Omega]$

Specific Resistance Eq.

$$\frac{\Delta Q}{s} = \rho \frac{I}{A}$$

$\left[\frac{V}{m}\right]$ $\left[\frac{V}{A}m\right]$ $\left[\frac{A}{m^2}\right]$

$[\Omega m]$

Conductivity Eq.

$$\frac{I}{A} = \frac{1}{\rho}\frac{\Delta Q}{s} = C \frac{\Delta Q}{s}$$

$\left[\frac{A}{m^2}\right]$ $\left[\frac{1}{\Omega m}\right]$ $\left[\frac{V}{m}\right]$

How do we measure conductivity? 11. 3. 2007

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Conductors A and B in series

$$I_A = I_B$$

$$(Q_3 - Q_2)\, C_B = (Q_2 - Q_1)\, C_A$$

$$C_A = \frac{Q_3 - Q_2}{Q_2 - Q_1}$$

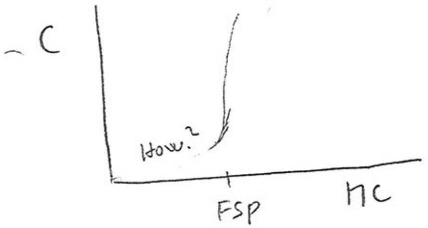
Some Specific Resistances

Dry Wood 10^{12} Ωm

Distilled Water $5 \cdot 10^{3}$ Ωm } $C = C(mc)$

$\Rightarrow$ specific conductivities $10^{-12} \frac{1}{\Omega m}$

$2 \cdot 10^{-4} \frac{1}{\Omega m}$



12.3. 2007

(12)

Signal Processing

Signal generator

Input transducer

Signal modifier

Output transducer

Displacement due to force

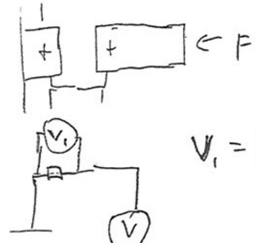
Electrical signal

Conversion to linear Voltage ± 5V

Conversion to Newtons

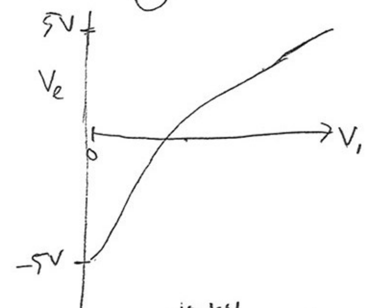
1°

2°

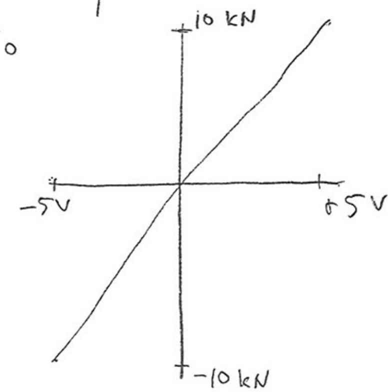


$V_1 = R_1 I$

3°



4°



Eventual problems

Signal generator displacement

-> specimen displacement

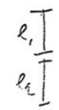
Finite Signal generator stiffness

Finite mounting system stiffness

Signal dampening

Temperature-dependence

Hysteresis



$$\delta = \delta_1 + \delta_2$$

$$= \frac{F}{k_1} + \frac{F}{k_2}$$

$F = k_1 \delta_1 = k_2 \delta_2$

Effective K:

$$K_e = \frac{F}{\delta} = \frac{F}{\delta_1 + \delta_2}$$

$$= \frac{F}{\frac{F}{k_1} + \frac{F}{k_2}} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

Solutions/Actions

Calibration

- Full range
- Zero offset

Temperature compensation

Signal Generator Displacement Compensation

13.3.2007 (13)

Noise

Signal-to-Noise-Ratio $\frac{S}{N} = \frac{\text{Signal Amplitude}}{\text{Noise Amplitude}}$

-> Detection limit

Fundamental Noise

Thermal - thermal movement of charge carriers

Shot - Impacts by individual charge carriers

Flicker - (?)

Environmental Noise

Electric & magnetic fields

Radiation

Mechanical Vibration

Others...

Car -> Heart rate meter

{Subway -> computer screen
(mobile phone ↗

Solutions - Actions

$\frac{S}{N} \uparrow$

Filtering

Integration - Boxcar
- Ensemble Averaging
- Moving-Average smoothing
- Weighed Moving-Average Smoothing

13.3.2007 (14)

Dirac δ

$$\int f(t)\,\delta(t-a)\,dt = f(a) \qquad \delta(t) = 0 \text{ for } t \neq 0$$

Fourier Transform

$$\mathfrak{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \quad \rightarrow \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(u)\, e^{-i\omega u}\, du$$

Inverse Fourier Transform

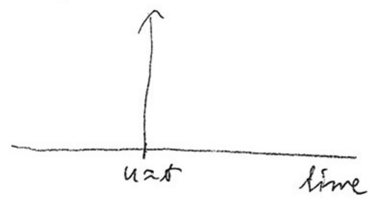
$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathfrak{F}(\omega)\, e^{i\omega t}\, d\omega$$

$$= \int_{-\infty}^{\infty} du\, f(u) \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega(t-u)}\, d\omega \right\}$$

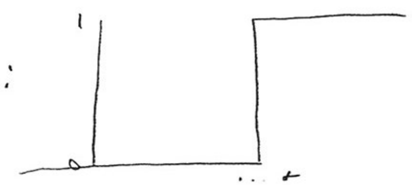
$$\Rightarrow \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega(t-u)}\, d\omega \right\} = \delta(u-t)$$

Euler Identity $e^{i\omega(t-u)} = \cos[\omega(t-u)] + i \sin[\omega(t-u)]$

=> ~~Inverse~~ Fourier Transform compresses a whole spectrum of harmonic waves into a narrow range $u \approx t$ in the time domain!



Cumulative Distribution:



$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{F}(\varrho)\, e^{i\varrho t}\, d\varrho$$

$$\mathcal{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} d\varrho\, \mathcal{F}(\varrho) \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{it(\varrho-\omega)}\, dt \right\}$$

$$\Rightarrow \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{it(\varrho-\omega)}\, dt \right\} = \delta(\omega - \varrho)$$

$\Rightarrow$ (Inverse) Fourier transform compresses a whole spectrum of functions into a narrow range $\omega \simeq \varrho$ in the frequency domain!
of the time domain

Spectroscopy

- Monitoring a spectrum of something

- Affected by interaction with matter

- Electromagnetic radiation
- Mechanical waves
- Particles

$\boxed{\text{Absorption Spectroscopy} \\ \text{Emission Spectroscopy} \\ \text{Scattering Spectroscopy}}$

Photon Energy $E = h\nu = \frac{hc}{\lambda}$
Discrete Energy levels
$\rightarrow$ Oscillations between states
- several modes of vibration

Classical Wave Equation $\frac{\partial^2}{\partial z^2} A = \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2}$

One Solution : $A = A_0 \cos(kz - \omega t)$

Verification:

$\frac{\partial}{\partial z} A = -k A_0 \sin(kz - \omega t)$ $\qquad \frac{1}{c^2} \frac{\partial A}{\partial t} = \frac{-\omega}{c^2} A_0 [-\sin(kz - \omega t)]$

$\frac{\partial^2}{\partial z^2} A = -k^2 A_0 \cos(kz - \omega t)$ $\qquad \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\frac{\omega^2}{c^2} A_0 \cos(kz - \omega t)$

$$\Rightarrow \boxed{k^2 = \frac{\omega^2}{c^2}}$$

Intensity $\propto$ (Field density)^2 $= A^2 = A_0^2 \cos^2(kz - \omega t)$

Frequency spectrum of Intensities $A^2(\omega) = \mathcal{F}(A^2(t))$

Lognitudinal Mechanical Waves

(17) 21.3.2007

$I = F\Delta t$

$P = mv = m\frac{\Delta s}{\Delta t}$

Fluid in a tube

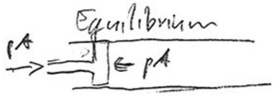
Momentum - Impulse theorem

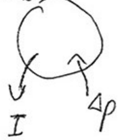
$F\Delta t = m\frac{\Delta v}{\Delta t}\Delta t$

$\Delta P = m\Delta v$

$\Delta P + I = 0$

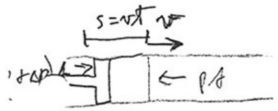
$\Delta P = -I$





Additional pressure Δp applied for time Δt

$\Rightarrow$ Acceleration of a Fluid Element of length $s = v\Delta t$, v = velocity of the frontier of the element



$\vec{v}_z$ = velocity of the piston

$\approx$ velocity of the element

Impulse by the liquid element: $I = -A\Delta p\,\Delta t$

Momentum change by the liquid element: $\Delta P = (\rho v\Delta t A)\,v_z$

Fractional volume change of the liquid element: $\frac{\delta V}{V} = -\frac{v_z\Delta t A}{v\Delta t A} = -\frac{v_z}{v}$

Bulk modulus $\equiv \frac{-\Delta p}{\Delta V/V} = B$

$\Rightarrow \Delta p = -B\frac{\Delta V}{V} = B\frac{v_z}{v}$

$\Delta P = -I$

$\rho v\Delta t A v_z = A\Delta p\,\Delta t$

$\rho v v_z = B\frac{v_z}{v}$

$v^2 = \frac{B}{\rho} \Rightarrow \boxed{v = \sqrt{\frac{B}{\rho}}}$

Uniaxial strain

Uniaxial stress:

$\Delta p = -E\varepsilon = -E\frac{\Delta l}{l}$

$\frac{\Delta l}{l} = -\frac{v_z\Delta t}{v\Delta t} \Rightarrow \Delta p = E\frac{v_z}{v}$

$\Delta P = -I$

$(\rho v\Delta t A)v_z = A\Delta p\,\Delta t$

$\rho v v_z = E\frac{v_z}{v} \Rightarrow \boxed{v = \sqrt{\frac{E}{\rho}}}$

Intensity of longitudinal wave

21.3.2007 (18)

Differential work $dW = \frac{\partial W}{\partial x_1}dx_1 + \frac{\partial W}{\partial x_2}dx_2 + \frac{\partial W}{\partial x_3}dx_3$

$= F_1\,dx_1 + F_2\,dx_2 + F_3\,dx_3$

$= F_i\,dx_i$

Force in one dimension

$\Rightarrow dW = F_1 dx_1 = F dx$

Power $\frac{dW}{dt} = \frac{F dx}{dt} = F\frac{dx}{dt} = Fv$

$\rightarrow Fv_z$

$\frac{dW}{dt} = Fv_z = \Delta p A v_z = B\frac{v_z}{v}Av_z$

$= B\frac{v_z^2}{v}A$

$v_z \equiv \frac{d\Delta}{dt}$

Intensity $= \frac{\text{Power}}{\text{Area}} = \frac{dW}{A\,dt} = B\frac{v_z^2}{v} = B\left(\frac{d\Delta}{dt}\right)^2\frac{1}{\sqrt{\frac{B}{\rho}}} = \sqrt{B\rho}\left(\frac{d\Delta}{dt}\right)^2$

Say $\Delta(x,t) = A_\Delta\sin(\omega t - kz) = A_\Delta\sin\omega(t - \frac{z}{v})$

$\omega = 2\pi f = \frac{2\pi}{\tau}$

$k = \frac{\omega}{v} = \frac{2\pi}{\lambda}$

$\frac{d\Delta}{dt} = A_\Delta\,\omega\cos(\omega t - kz)$

$\left(\frac{d\Delta}{dt}\right)^2 = A_\Delta^2\omega^2\cos^2(\omega t - kz)$

Intensity $= \sqrt{B\rho}\,A_\Delta^2\omega^2\cos^2(\omega t - kz)$

Average Intensity $= \frac{1}{\tau}\int_0^\tau \sqrt{B\rho}\,A_\Delta^2\omega^2\cos^2(\omega t - kz)\,dt = \frac{1}{2}\sqrt{B\rho}\,A_\Delta^2\omega^2$

$= \frac{1}{2}\rho v A_\Delta^2\omega^2$

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How to determine A_Δ ?

$$\Delta p = B\frac{v_z}{v} = B\frac{d\Delta}{dt}\frac{1}{v}$$

If $\frac{d\Delta}{dt} = A_\Delta \omega \cos(\omega t - kz)$

$$\Delta p = \frac{B}{v} A_\Delta \omega \cos(\omega t - kz)$$

$$\Rightarrow A_p = \frac{B}{v} A_\Delta \omega \Rightarrow A_\Delta = A_p \frac{v}{B\omega} = A_p \frac{1}{\sqrt{BS}\,\omega}$$

-> Measure pressure amplitude, compute displacement amplitude!

22.3.2007

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Thermal transitions

- changes in thermal properties

First-order transition

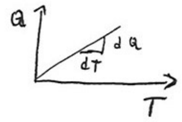
- change in heat capacity
- latent heat involved

Second-order transition

- change in heat capacity only

Heat: thermal energy [J] Q

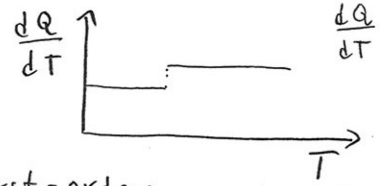
Heat Capacity: $\frac{dQ}{dT}$



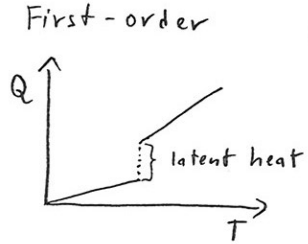
Heat flow rate $\frac{dQ}{dt}$

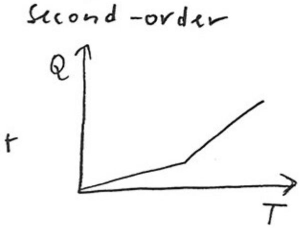
Temperature change rate $\frac{dT}{dt}$

Thermal transition

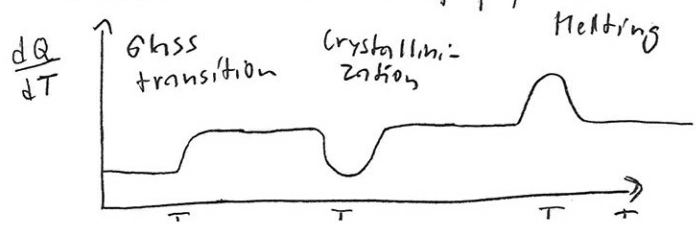


$$\frac{dQ}{dT} = \frac{dQ/dt}{dT/dt}$$





Thermal transitions of polymers



Let us produce heat in a resistor:

Potential difference $\Delta P = P_2 - P_1$ $\quad [V] = \left[\frac{J}{C}\right]$

Current I $\quad [A] = \left[\frac{C}{s}\right]$

Power $\Delta P\, I \rightarrow \boxed{\frac{dQ}{dt}}$ $\quad \left[\frac{J}{s}\right]$

Work $\int \Delta P\, I\, dt$

-> dissipated as heat

Calorimetry:

Measurement of Heat flows

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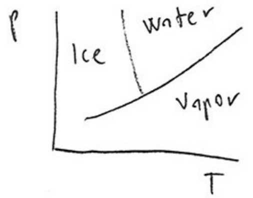
An application: melting temperature spectrum of cell wall water

Clausius - Clapeyron

$$\frac{dp}{dT} = \frac{\Delta H}{T \Delta V} = \frac{H_2 - H_1}{T(V_2 - V_1)} < 0!$$



$$dp = \frac{\Delta H}{\Delta V} \frac{dT}{T}$$

$$p = \frac{\Delta H}{\Delta V} \ln T + C$$

$$\ln T = p \frac{\Delta V}{\Delta H} + C_2$$

$$T_m = C_3\, e^{p \frac{\Delta V}{\Delta H}} = C_3\, e^{-p \left|\frac{\Delta V}{\Delta H}\right|}$$

Kelvin Eq. II

$$p(r) = p_\infty\, e^{\frac{V 2 \gamma}{R T r}}$$

substitute

$$\frac{T_m}{T_0} = \frac{e^{-e^{\frac{V2\gamma}{RTr}} p_\infty \left|\frac{\Delta V}{\Delta H}\right|}}{e^{-p_\infty \left|\frac{\Delta V}{\Delta H}\right|}} = e^{p_\infty \left|\frac{\Delta V}{\Delta H}\right| (1 - e^{\frac{V2\gamma}{RTr}})}$$

$$\ln \frac{T_m}{T_0} = p_\infty \left|\frac{\Delta V}{\Delta H}\right| \left(1 - e^{\frac{V2\gamma}{RTr}}\right)$$

$$e^{\frac{V2\gamma}{RTr}} = 1 - \frac{\ln \frac{T_m}{T_0}}{p_\infty \left|\frac{\Delta V}{\Delta H}\right|}$$

$$\boxed{r = \text{function}\,(T_m)}$$

$$\frac{V2\gamma}{RTr} = \ln (\quad)$$

$$\boxed{r = \frac{V 2 \gamma}{RT \ln \left(1 - \frac{\ln \frac{T_m}{T_0}}{p_\infty \left|\frac{\Delta V}{\Delta H}\right|}\right)}}$$

w. some assumptions, simplified Gibbs-Thompson Eq.

$$\boxed{r = - \frac{V \gamma}{\Delta H_m \ln \frac{T_m}{T_0}}}$$

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Melting Temperature Spectrum

→ pore size distribution

T [°C]	D [nm]
-30	1,4
-10	4,2
-5	8,6
-2,5	17
-1,2	36
-0,6	72
-0,3	144
-0,2	216
-0,1	433

Mass of freezing water in pores of diameter between D_1 and D_2

$$[m_{FW}]_{D_1 D_2} = \frac{1}{\Delta H_m} \int_{T_1(D_1)}^{T_2(D_2)} \frac{dQ}{dT}\, dT = \frac{1}{\Delta H_m} \int_{t_1(T_1)}^{t_2(T_2)} \frac{dQ}{dt}\, dt$$

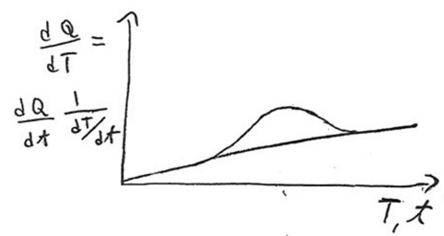
$$\Delta H_m \approx 333 \ \mathrm{J/g}$$

How about the effect of Heat Capacity?

$$Q = Q_T(\text{transition}) + Q_S(\text{temperature increment}) \Rightarrow Q_T = Q - Q_S$$

$$[m_{FW}]_{D_1 D_2} = \frac{1}{\Delta H_m} \int_{T_1(D_1)}^{T_2(D_2)} \left(\frac{dQ}{dT} - \frac{dQ_S}{dT}\right) dT \approx \frac{1}{\Delta H_m} \left[\int_{T_1}^{T_2} \frac{dQ}{dT}\, dT - \frac{dQ_S}{dT}(T_2 - T_1) \right]$$

How to determine Heat Capacity $\frac{dQ_S}{dT}(T)$?



Non-Freezing Water

$$NFW = \left[m_w - [m_{FW}]_{-\infty} \right] \frac{1}{m_0}$$

Total Cell Wall Water

$$m_{cw} = NFW - [m_{FW}]_{-\infty\, T_0^-} \approx FSP \cdot m_0 \ ?$$

Uniaxial {Kelvin / Thermoelastic Eq.

$$T l \alpha \, dF = dQ$$

T temperature
l length
α thermal expansion Eq.
F Force
Q Heat

Black-Body Radiation

$$I_{\lambda b} = \frac{2\pi h c^2}{\lambda^5 \left[e^{\frac{hc}{\lambda k T}} - 1 \right]}$$

Grey-Body Radiation

$$I_\lambda = \varepsilon I_{\lambda b}$$

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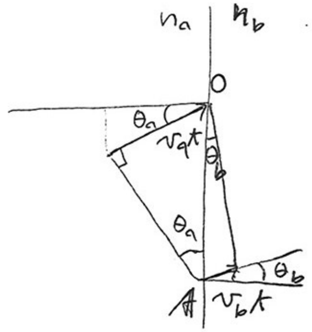
Snell's Law of Refraction

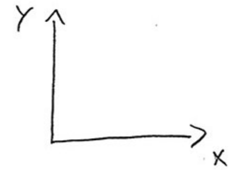


$$\sin\theta_a = \frac{v_a t}{AO} \Rightarrow AO = \frac{v_a t}{\sin\theta_a}$$

$$\sin\theta_b = \frac{v_b t}{AO} \Rightarrow AO = \frac{v_b t}{\sin\theta_b}$$

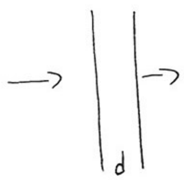
$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{v_a}{v_b} = \frac{c/v_b}{c/v_a} = \frac{n_b}{n_a}$$

Birefringence: Anisotropic Index of Refraction



$$n_x = c/v_x \neq n_y = \frac{c}{v_y}$$

Passage through a layer d



PassageTime difference $\quad t = \frac{d}{v}$

$$\Delta t = d\left(\frac{1}{v_x} - \frac{1}{v_y}\right)$$

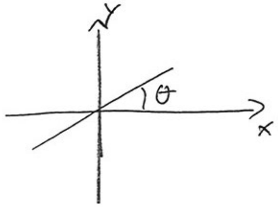
Path Difference (after crossing the layer)

$$p = \Delta t c = d\left(\frac{c}{v_x} - \frac{c}{v_y}\right) = d(n_x - n_y)$$

Phase Difference

$$\delta = 2\pi \frac{p}{\lambda} = 2\pi \frac{d}{\lambda}(n_x - n_y)$$

Apply linearly polarized light



$A = A_0 \sin \omega t$

$A_x = A \cos\theta = A_0 \cos\theta \sin(\omega t)$

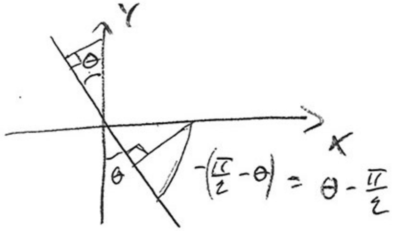
$A_y = A \sin\theta = A_0 \sin\theta \sin(\omega t)$

After passage of layer d

$A'_x = A_0 \cos\theta \sin(\omega t)$

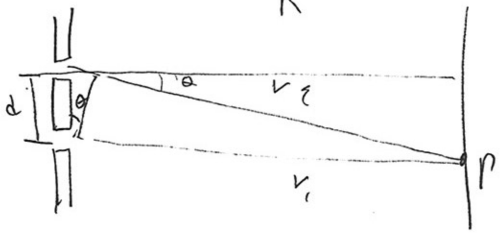
$A'_y = A_0 \sin\theta \sin(\omega t + \delta)$

Projections to Analyzer, perpendicular to polarizer



$$S = \cos\theta A'_y - \cos\left(\theta - \frac{\pi}{2}\right) A'_x$$
$$= \cos\theta \sin\theta \sin(\omega t + \delta) A_0$$
$$- \sin\theta \cos\theta \sin(\omega t) A_0$$
$$= \sin\theta\cos\theta \left[\sin(\omega t + \delta) - \sin(\omega t)\right]$$
$$= \frac{A_0}{2} \sin(2\theta) \left[\sin(\omega t + \delta) - \sin(\omega t)\right]$$
$$\left[\sin\left(\omega t + 2\pi \frac{P}{\lambda}\right) - \sin(\omega t)\right]$$
$$\left[\sin\left[\omega t + 2\pi \frac{d}{\lambda}(n_x - n_y)\right] - \sin(\omega t)\right]$$

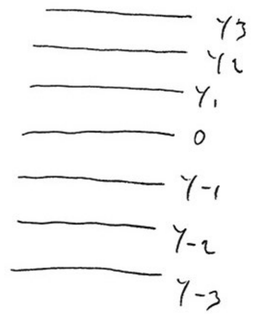
Interference → Diffraction



Positive Interference: $d \sin\theta = m\lambda$

Destructive Interference: $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$ $m = 0, \pm 1, \pm 2, \ldots$

Interference Pattern:



$$\frac{Y_m}{R} = \tan\theta_m \approx \sin\theta_m$$

$$Y_m \approx R \sin\theta_m = R \frac{m\lambda}{d}$$